The automation of generalized curves method presentation on the map at any scales

Tadeusz Chrobak, Professor of AGH
Faculty of Mining Surveying
and Enviormrntal Engineering
AGH University of Science and Technology
Al. A. Mickiewicza 30
30 – 059 Kraków, Poland
e – mail: tchrobak@uci.agh.edu.pl

Abstract

As elaborated map scale is decreasing, open and closed (defining areal objects) curves simplifying process is characterized by the points loss untill the border state, i.e. the curves removing. This elimination takes place when two points of the open and iregular closed curve or three points of regular closed curve (such as dwelling-houses) after simplification remains. Some occuring in the curves simplification process caused by the scale changing distinctive intermediate stages of the results map presentation after generalization L. Ratajski (1989) called the generalization thresholds.
The article's main subject is to define simplified curves parameters, which will determine the generalization thresholds for map edited at any scale. These are such as: broken curve simplification, curves simplification with smoothing, symbolization and elimination. To obtain the answers the author used: an objective curves simplifying method and an mathematical statistics properties.

Using mathematical statistics in the curves simplifying process

In the curves simplifying process conducted by the objective method (Chrobak, 2003) points elimination depends on their hierarchy (resulting from relative extrema) and drawing recognizability, that is objective factors. The recognizability measure is defined as the shortest length of an elementary, made of three simplified curve points, triangle side. All the simplified curve points participate in this investigation.

Assuming that an original curve is a reality reflection its points describe the most probable curve. However, the curve accuracy measure after simplification are the shortest distances Δl_i, i = 1, 2, 3, … between remaining and rejected points. These distances are apparent errors of simplified curve shape evaluation. According to the error propagation law, processed curve accuracy is qualified by the mean error – m_{upr}, which has a form as below:

$$m_{upr} = \pm\sqrt{(\Delta l_i)^2 / (n-1)} \qquad (1)$$

where:

$\Delta l_i = \pm\sqrt{\Delta x_{ij}^2 + \Delta y_{ij}^2}$

$\Delta x_{ij} = x_{pi} - x_{uj}$,

$\Delta y_{ij} = y_{pi} - y_{uj}$,

n – the number of rejected points,

n -1 – the number of segments,

p_i – remaining point,

u_j – rejected point.

In the curve simplifying metod the number of rejected points and the mean error m_{upr} doesn't depend on a map's editor, therefore the process results maintain statistic distribution properties. Probability density function defines expected value – EV(X) and results statistic dispertion, as standard deviation is (see Figure 1). Expected value of curve (both open and closed) simplifying is defined by the number of well-ordered points describing its generalized shape. The dispertion measure is defined by the standard deviation – σ (curves simplifying process mean error) – Figure 1.

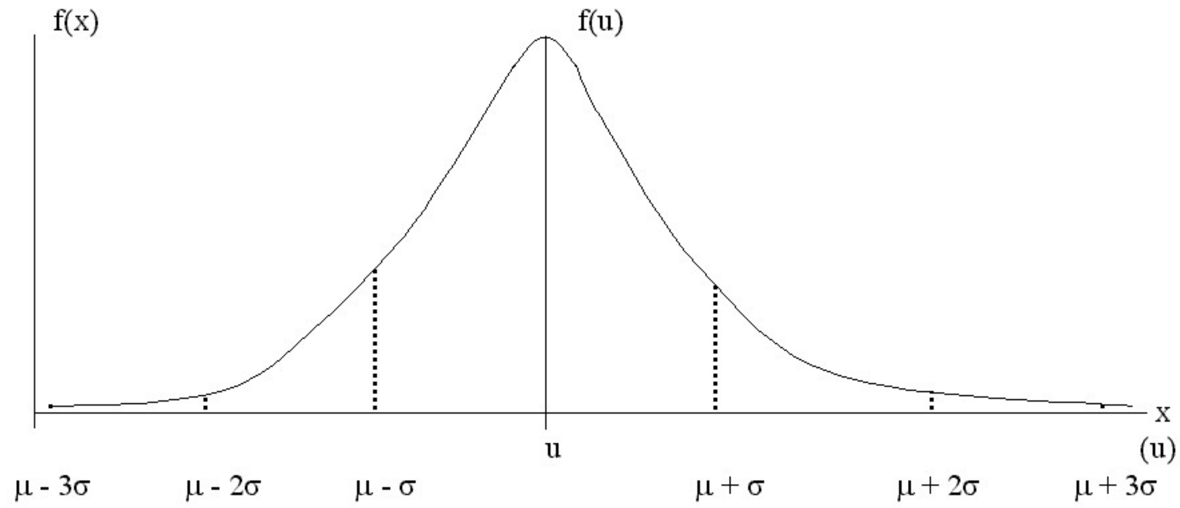

Figure. 1 Probability density function of normal – f(x) and normalized – f(u) distribution

According to normal statistic distribution interpretation probability of random variable X obtaining with uncertainty corresponding with 1σ (mean error – m_{upr}) equals 68%. Probability of random variable X obtaining with uncertainty corresponding with 2σ is 90% and with 3σ respectively 95%.

Thus, random variable expected value – EV(X) (defined by the number of points: n_0 – original curve points, n_i – generalized curve points, c – proces invariable points) and σ – standard deviation are the parameters of choosing presentation method for displaying open and closed curves generalized by the Chrobak method while changing the map scale.

Parameters defining threshold of simplifying curves that are presented on the map

Generalization thresholds of simplified open and closed curves are defined by dependance:

$$\left(100\frac{n_i - c}{n_0} - k\sigma\right) = \min \in [-5, 5), \quad [\%] \tag{2}$$

where:

n_0 – the number of the original curve points,

n_i – the number of points after generalization,

c – the number of process invariable points,
k – the factor , where k = 1, 2, 3,
1σ – standard deviation with probability equals 68%,
2σ – standard deviation with probability equals 90 %,
3σ – standard deviation with probability equals 95 %.

The curve generalization thresholds results from formula (2):

- open or closed broken curve for k = 1,
- open or closed smoothed curve for k = 2,
- open curve elimination for k = 3,
- symbol (e.g. circle with x mm diameter instead of closed curve) for k = 3.

Assumed for formula (2) ranges limits the number of generalized curve points, which retains the original curve shape (with k = 1) with accuracy defined by the GUGiK standard (GUGiK means the Main Geodesy and Cartography Office):

$$\sigma = m_{upr} = \pm 0.7\, M_{00} \quad [\text{mm}] \tag{3}$$

where:
M_{00} – the scale denominator

The objective curve simplifying method complies with the condition of the formula (3), because the shortest recognizability triangle side length of the drawing receive values of 0,5 – 0,6 M_{00}.

Examples

Besides rejecting there is also points adding in curves simplifying objective method, so that transformed curve remain similar to its original shape. Adding new points takes place in finite scale series range in the original scale nearest neighbourhood (see Figure 2).

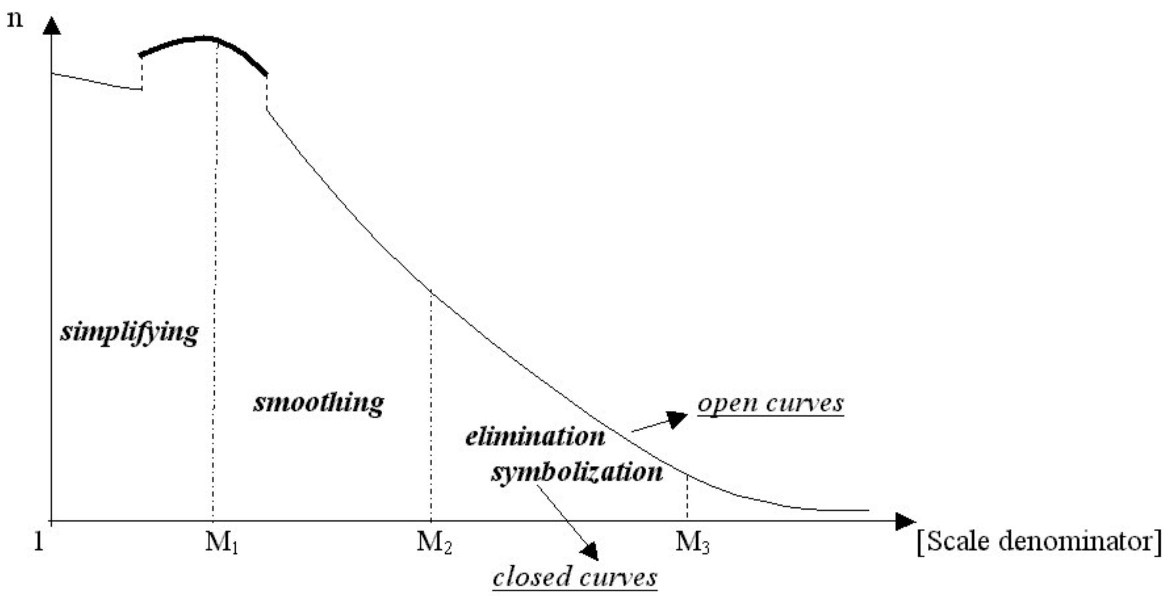

Figure 2 Rejecting points while the map scale is changing

The results of changing the points number – n at scale denominator – M due to curves simplifying objective method has been compared with the generalization threshold defined by the formula (2). The results shown in Table 1 points out the conformity of the scales with the maximum added points and generalization threshold given by the formula (2). What proves, that the standard deviation is the measure that is indispensable when specifying generalization thresholds (for curves simplified with the objective method) due to scale changing.

Table 1 Examples

No.	Scale	n0/c	Added points	Rejected points	ni	min	k=1	k=2	k=3
1	2	3	4		5	6	7	8	9
1									
	1:1000	133/16	0	0	133				
	1: 2000		1	1	133				
	1: 3000		1	1	133				
	1: 4000		2	2	133	-20	No		
	1: 5000		16	52	97	7	Yes		
	1: 6000		12	72	73	25			
	1:7000		5	83	55				
	1:8000		1	92	42				
	1:9000		0	94	39	17			
	1:10000		1	105	29	0		Yes	
	1:25000				16				Yes
	1:50000				16				
	1:100000				16				
2									
	1:1000	157/16	0	0	157				
	1: 2000		0	0	157				
	1: 3000		0	0	157				
	1: 4000		6	6	157	21	No		
	1: 5000		40	74	123	0	Yes		
	1: 6000		13	90	80	-25	No		
	1:7000		2	94	65				
	1:8000		3	112	48				
	1:9000		0	112	45	8		No	
	1:10000		1	121	37	3		Yes	
	1:25000		0	141	16				No
	1:50000		0	141	16				
	1:100000		0	141	16				
3									
	1:1000	157/16	0	0	157				
	1: 2000		0	0	157				
	1: 3000		0	0	157				
	1: 4000		6	6	157	21	No		
	1: 5000		40	74	123	0	Yes		

	1: 6000		13	90	80	-25	No		
	1:7000		2	94	65				
	1:8000		3	112	48				
	1:9000		0	112	45	8		No	
	1:10000		1	121	37	3		Yes	
	1:25000		0	141	16				No
	1:50000		0	141	16				
	1:100000		0	141	16				
6									
	1:1000	155/20	0	0	155				
	1: 2000		0	0	155				
	1: 3000		0	0	155				
	1: 4000		0	0	155				
	1:5000		37	58	134	5	Yes		
	1: 6000		21	87	89	24	No		
	1: 7000		0	87	68				
	1: 8000		0	109	46				
	1: 9000		0	109	46				
	1: 10000		1	117	39	2		Yes	
	1: 25000		0	135	20				No
	1: 50000		0	135	20				
	1:100000		0	135	20				

Conclusions

The open and closed curves simplified by the objective method generalization thresholds can be defined by the method that is based on the mathematical statistics. The conformity of generalization thresholds scales and the maximum of added points scale within 1σ range proves the threshold defining correctness, because added points are used to obtain optimal conformity of generalized curve and its original shape.

References

Chrobak T., The method of curve simplification in case of map scale alteration 10th International Conference, GI/GIT Theory and Practice, Bring them Together, Ostrawa, 27-29.01.03.

Szydłowski H., Teoria pomiarów, PWN Warszawa 1991.

Ratajski L., Metodyka kartografii społeczno – gospodarczej, Warszawa - PPWK 1989.