DETECTION OF SINGULARITIES BY USING DEM TOOLS

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Abstract. Detection of Singularities by Using DEM Tools. This paper describes methods for locating singular points of the digital elevation maps and its structural maps. Search for singularities is consequently search for the local disturbances of investigated phenomenon. Singularities are important points on the surface. They can help to find faults on elevation data, or at visualization of the data. Also, description of the structural land surface parameters (LSPs) is provided at the paper. LSPs such as: gradient, slope, aspect and curvatures and some characteristics of them are defined and described as well.

Keywords: singularity, geomorphometry, land surface parameters, gradient, slope, aspect, curvature

1 Introduction

In matematics, a singularity is in general a point at which a given mathematical object is not defined, or a point of an exceptional set where it fails to be well-behaved in some particular way, such as differentiability [9]. Search for singularities in scalar field or its isoline field is consequently search for the local disturbances of investigated phenomenon. Localization of the disturbances on the investigated phenomenon is equally interesting from physical as well as from cartographic point of view. Knowledge of singularities in GIS is coessential for its own visualization. Surface singularities are prominent landmarks and their detection, recognition and classification is a crucial step in computer vision and 3D graphics [3].

The information concerning the singular points is more or less contained in the input digital elevation datasets, in the grid or irregular points data files. However, working with this information smoothing often occurs, thus minor singular areas are eliminated, as well as local disturbances respectively. In the case of output regular grids this effect is even more supported by its regularity. In the case of the irregular although representative points, construction of the triangulated irregular network (TIN) can be corrupted [7].

The visual presentation itself, no matter what the form it is (grid or isolines), emphasizes this problem depending on selected hypsometric interval. The situation is the most evident in extensive computer presentations using colored hypsometry method where number of colors and its shades distinguishable by human eye can clearly prove not to be enough for visualization of all surface forms in zoomed domain. Using analogue cartography, map consisting of contour lines is being added by cartographical symbols of saddle, hill or pit elevation points. Visualization of these points must be preceded by their identification. In the GIS the methods for locating singular points of elevation should be a part of implemented DEM tools. The aim of this paper is to present that it does not need to be a very complicated problem.

2 Detection of singularities

We consider the scalar field which represents the adequately smooth surface. That means, the surface without sharp edges which can be expressed by the single-value, continuous and differentiable scalar function $s = f_s\ (x, y)$. Using simplified definition, if this equation is equation in explicit form for surface expressed by real-valued height function $\boldsymbol{f_s} : \mathbb{M} \rightarrow \mathbb{R}$ on an image which is a smooth manifold $\mathbb{M}$,[1] and has only non-degenerate critical or singular points then it is a Morse function[2]. The critical or singular points of the Morse function are points of maxima, minima or points of double saddle[3]. Thus, its identification can be done based on topological characteristics of a surface.

An algorithm for extraction of the singular points in grid can be based on the method of eight-neighbor points [1], [14], called also 3x3 moving kernel method [5], which compares the height of investigated point with the height of eight-adjacent neighbors. A peak is given by a point that is higher than all other points in its neighborhood. A pit is given by a point that is lower than all other points in its neighborhood. Saddle points are the hardest to identify in DEM and there are no algorithms found for locating saddle points in literature [5]. For the saddle point it is generally valid that there exist four point couples-neighbours in its neighborhood, such that one point of point couple must be higher and the second one lower than the investigated point.

We can avoid usage of this algorithm, if we know partial derivations of function $f_s\ (x, y)$. Based on statements from the second note it is shown, that for identification of the singular points of function $f_s\ (x, y)$ we need to know its points in which the first order partial derivatives are as follows

$$\frac{\partial f_s}{\partial x} = \frac{\partial f_s}{\partial y} = 0.$$

If the function $f_s\ (x, y)$ is a Morse function then, the signs of its second order partial derivatives are the only necessary information for classification of its singular points. If the function $f_s\ (x, y)$ is not Morse function to distinguish non-degenerated and degenerated critical points we need to know value of its second order partial derivatives. Example of a degenerated critical point is inflection point of landslide, in which valley line is turning into crest line, limiting case of double saddle point and triple saddle point called also known as monkey saddle (saddle for a monkey requires two depressions for the legs and one for the tail) (Fig. 1), but as well quadruple and higher-order saddle point. Inflection point of landslide, limiting case of double saddle point, triple and higher saddle points are unstable points and may be removed by a slight deformation of land surface.

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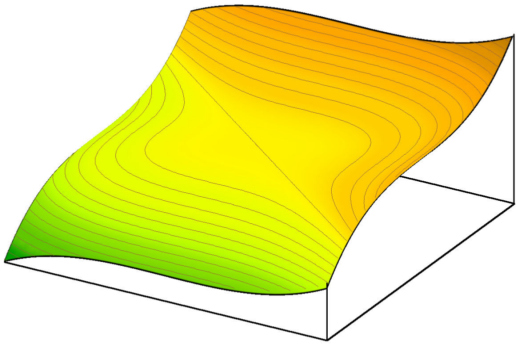
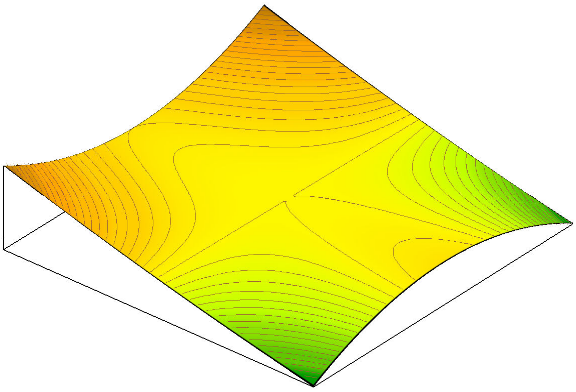
[1] Hence, $\boldsymbol{f_s}$ is an orthogonal projection with respect to the z axis and two-dimensional manifold $\mathbb{M} \subseteq \mathbb{R}^3$ defined as $\mathbb{M} = \{(x, y, z)\} : z = \boldsymbol{f_s}\,(x, y)\}$.

[2] A point (x_o, y_o) is critical or singular point of function $f_s\ (x, y)$ if gradient ∇s at the point (x_o, y_o) vanished, i.e. $\nabla s = 0$. Singular point is non-degenerate if the Hessian matrix $\nabla^2 s$ of second order partial derivatives of function $f_s\ (x, y)$ is non-singular, than it has nonzero determinant.

[3] Let point (x_o, y_o) is non-degenerate critical point and Hessian is determinant of Hessian matrix, which is product of its two eigenvalues. If the Hessian is positive definite at point (x_o, y_o), then function $f_s\ (x, y)$ attains a local extreme at point (x_o, y_o) and point (x_o, y_o) is isolated singular point. Eigenvalues then are both positive for pit, or both negative for pick. If the Hessian is negative definite at point (x_o, y_o), then function $f_s\ (x, y)$ attains a saddle at point (x_o, y_o) and two eigenwalues of Hessian matrix have different signs.



a)



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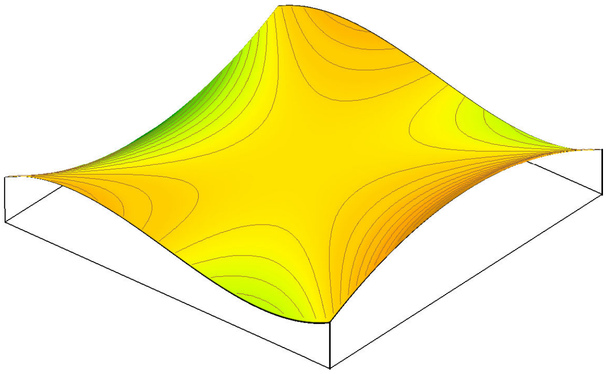

c)

Fig. 1. Samples of degenerate critical points area – a) inflection point of landslide, b) limiting case of saddle point, c) triple or monkey saddle point

3.1 Structural scalar fields

Methods mentioned above for locating singularities generated by grid or points data file used the information already contained in the scalars values of these datasets, or the partial derivatives of scalar fields calculated from the datasets. The worse situation is in identification of the singularities of the *structural* scalar fields.

At the present time, the geomorphometry uses the particular local land surface parameters (LSPs) such as: magnitude (length) of the two-dimensional gradient vector usualy denoted as $|\nabla s|$ or $|\boldsymbol{grad}\, s|$,[4] slope angle γ_N, aspect angle A_N, profile curvature $(K_N)_n$, plan and tangential curvature K_r and $(K_N)_t$. These LSPs create abstract scalar fields which are the structural scalar fields of the original scalar field. For the scalar field of altitude we can write function $s = f_s\,(x, y)$ in the form of $h = f_h\,(x, y)$ or simplified $h = h\,(x, y)$. In accordance to the papers [10], [8], [5] and [12] LSPs can be divided by criterion of highest order partial derivative of function $h = h\,(x, y)$ contained in formulas that express scalar fields of these LSPs. According to them, magnitude of the altitude gradient $|\nabla h|$, slope γ_N and aspect A_N are first order LSPs and curvatures $(K_N)_n$ and K_r and $(K_N)_t$ are second order LSPs.

[4] Magnitude of vector is reffered to as the vector norm. In many texts, vector norms are indicated by a double bar, e.g., $\|\nabla s\|$ is the norm or magnitude of the gradient vector ∇s .

In paper [4] scalar fields of slope γ_N and aspect A_N (opposite direction of normal to contour line or gradient vector measured in the plane of x and y co-ordinates) expressed based on the first order partial derivatives of function $h = h\,(x, y)$ are considered as the first and second structural scalar field of the original scalar field of altitudes. By the derivation of functions of these structural scalar fields in one from the two fundamental directions of surface, direction of normal to contour line and direction of tangent to contour line, results in structural fields of LSPs of second order. The result of derivation of structural fields of LSPs of second order in one from the two fundamental directions of surface are structural fields of LSPs third order, etc. Particularly LSPs formulas derived from the first and second structural scalar field in paper [4] symbols of S and A were used to express the first structural scalar field of slope γ_N and second structural scalar field of aspect A_N and symbols $_n$ and $_t$ to express the direction of derivation, lower index $_n$ normal direction and lower index $_t$ tangent direction.

Generally using these symbols for profile curvature following form can be shown:

$$(K_N)_n = S_n \cos S \equiv S_n \cos \gamma_N = -\frac{\partial \gamma_N}{\partial n} \cos \gamma_N ,$$

for plan curvature K_r respectively:

$$K_r \equiv A_t = \frac{\partial A_N}{\partial t}$$

and for tangential curvature $(K_N)_t$:

$$(K_N)_t = A_t \sin S \equiv A_t \sin \gamma_N = \frac{\partial A_N}{\partial t} \sin \gamma_N .$$

In paper [8] the scalar fields of slope γ_N and aspect A_N in position of the first and second structural scalar field were complemented by scalar field of gradient magnitude $|\nabla h|$. In such a way a number of theoretically possible and derived LSPs second and heigher orders was increased. For formulas of particular LSPs derived from scalar field of gradient magnitude, in paper [8] symbol D was used to express scalar field of gradient magnitude.

It is possible to derive formulas expressing LSPs of next order by using the described method for expression of structural scalar fields, function's derivation expressing LSPs of given order. By this method it is possible to express LSPs up to the order which depends on quality of function $h = h\,(x, y)$ which need to have continuous partial derivatives at least up to the previous order.

In accordance with mentioned above and set (11) in the paper [8], we can theoretically write the potential base of LSPs set in the form of:

$$\mathrm{G_{RF}} = \left\{ {}^{(0)}\mathrm{G_{RF}} , {}^{(1)}\mathrm{G_{RF}}, {}^{(2)}\mathrm{G_{RF}}, \ldots , {}^{(k)}\mathrm{G_{RF}} \right\},$$

when its subsets have the following form:

$${}^{(0)}\mathrm{G_{RF}} = \left\{ h \right\}$$

$$ {}^{(1)}\mathrm{G}_{\mathrm{RF}} = \Big\{ D \equiv |\nabla h| \equiv |\boldsymbol{grad}\; h| = \mathrm{tg}\,\gamma_N = \frac{\partial h}{\partial n} = \sqrt{\left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2} \,,\; S \equiv \gamma_N = \mathrm{arctg}\left(\frac{\partial h}{\partial n}\right), $$

$$ A \equiv A_N = \mathrm{arctg}\left(- \frac{\partial h}{\partial y} \Big/ - \frac{\partial h}{\partial x} \right) \Big\} $$

$$ {}^{(2)}\mathrm{G}_{\mathrm{RF}} = \Big\{ D_n = - \frac{\partial |\nabla h|}{\partial n}, D_t = \frac{\partial |\nabla h|}{\partial t}, S_n = - \frac{\partial \gamma_N}{\partial n}, (K_N)_n = S_n \cos \gamma_N \,, S_t = \frac{\partial \gamma_N}{\partial t}, A_t \equiv K_r = \frac{\partial A_N}{\partial t}, $$

$$ (K_N)_t = K_r \sin \gamma_N \,, A_n = - \frac{\partial A_N}{\partial n} \,, D_2 \Big\} $$

$$ {}^{(3)}\mathrm{G}_{\mathrm{RF}} = \Big\{ D_{nn} = - \frac{\partial D_n}{\partial n},\; D_{nt} = \frac{\partial D_n}{\partial t},\; D_{tt} = \frac{\partial D_t}{\partial t},\; D_{tn} = - \frac{\partial D_t}{\partial n},\; S_{nn} = - \frac{\partial S_n}{\partial n},\; S_{nt} = \frac{\partial S_n}{\partial t}, $$

$$ (K_N)_{nn} = - \frac{\partial (K_N)_n}{\partial n},\; (K_N)_{nt} = \frac{\partial (K_N)_n}{\partial t},\; S_{tt} = \frac{\partial S_t}{\partial t},\; S_{tn} = - \frac{\partial S_t}{\partial n},\; A_{tt} = \frac{\partial A_t}{\partial t},\; A_{tn} = - \frac{\partial A_t}{\partial n}, $$

$$ (K_N)_{tt} = \frac{\partial (K_N)_t}{\partial t},\; (K_N)_{tn} = - \frac{\partial (K_N)_t}{\partial n},\; A_{nn} = - \frac{\partial A_n}{\partial n},\; A_{nt} = \frac{\partial A_n}{\partial t} \Big\} $$

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$$ {}^{(k)}\mathrm{G}_{\mathrm{RF}} = \left[\, (s_k)_i \, \right]_{i=1}^{2^{(k+1)}} $$

Negative signs in second members of equations of LSPs, which were expressed by the normal direction derivative mean, that in contrast with uphill normal direction we take into consideration opposite orientation in flow direction.

3.2 Singularities of structural scalar fields

There are singular points of its structural scalar fields constrained at the singular points of the altitude scalar field. These points potentially can have different characteristics. For example, the singular points of scalar fields of particular LSPs can be degenerated critical points even if constrained to the non-degenerated critical points of original scalar field. This quality has for example the scalar field of aspect A_N or scalar fields of second or higher orders.

Not all singular points of structural scalar fields are constrained to the singular points of original scalar field. This means, that not all singular points of particular LSPs scalar fields have to be identical with the centre points of original grid or with node points of original regular as well as irregular geometric networks.

Zero isolines of the profile curvature passes through these singular points in scalar fields of slope angle γ_N and gradient magnitude $|\nabla h|$ and zero isolines of the plan or tangential curvature passes through these singular points in scalar field of slope aspect A_N [6].

If the following conditions are valid:

$$ \frac{\partial |\nabla h|}{\partial x} = \frac{\partial |\nabla h|}{\partial y} = 0 \;\vee\; \frac{\partial \gamma_N}{\partial x} = \frac{\partial \gamma_N}{\partial y} = 0 \,, $$

which is accomplished only when

$$\frac{\partial h}{\partial x}\frac{\partial^2 h}{\partial x^2} + \frac{\partial h}{\partial y}\frac{\partial^2 h}{\partial y \partial x} = 0 \;\wedge\; \frac{\partial h}{\partial x}\frac{\partial^2 h}{\partial x \partial y} + \frac{\partial h}{\partial y}\frac{\partial^2 h}{\partial y^2} = 0\,,$$

in accordance with paper [6] the structural scalar fields of gradient magnitude $|\nabla h|$ and slope angle γ_N derived from function $h = h\,(x, y)$ of original scalar field of altitudes have except the singular points constrained to the peak, pit and saddle points of function $h = h\,(x, y)$ also the singular points, for which the following conditions are valid:

1. $\frac{\partial h}{\partial x} = 0 \;\wedge\; \frac{\partial h}{\partial y} \neq 0 \;\Rightarrow\; \frac{\partial^2 h}{\partial y \partial x} = 0 \;\wedge\; \frac{\partial^2 h}{\partial y^2} = 0$ whereby $\frac{\partial^2 h}{\partial x^2} \neq 0 \;\vee\; \frac{\partial^2 h}{\partial x^2} = 0,$

2. $\frac{\partial h}{\partial x} \neq 0 \;\wedge\; \frac{\partial h}{\partial y} = 0 \;\Rightarrow\; \frac{\partial^2 h}{\partial x^2} = 0 \;\wedge\; \frac{\partial^2 h}{\partial x \partial y} = 0$ whereby $\frac{\partial^2 h}{\partial y^2} \neq 0 \;\vee\; \frac{\partial^2 h}{\partial y^2} = 0,$

3. $\frac{\partial h}{\partial x} \neq 0 \;\wedge\; \frac{\partial h}{\partial y} \neq 0 \;\Rightarrow\; \frac{\partial^2 h}{\partial x^2} = 0 \;\wedge\; \frac{\partial^2 h}{\partial y \partial x} = 0 \;\wedge\; \frac{\partial^2 h}{\partial y^2} = 0.$

Similarly, if the following condition is valid:

$$\frac{\partial A_N}{\partial x} = \frac{\partial A_N}{\partial y} = 0\,,$$

which is accomplished only when

$$\frac{\partial h}{\partial x}\frac{\partial^2 h}{\partial y^2} - \frac{\partial h}{\partial y}\frac{\partial^2 h}{\partial x \partial y} = 0 \;\wedge\; \frac{\partial h}{\partial x}\frac{\partial^2 h}{\partial y \partial x} - \frac{\partial h}{\partial y}\frac{\partial^2 h}{\partial x^2} = 0\,,$$

the structural field of slope aspect A_N derived from function $h = h\,(x, y)$ of original scalar fields of altitudes has except the singular points constrained to the peak, pit and saddle points of function $h = h\,(x, y)$ also the singular points, for which the following conditions are valid:

1. $\frac{\partial h}{\partial x} = 0 \;\wedge\; \frac{\partial h}{\partial y} \neq 0 \;\Rightarrow\; \frac{\partial^2 h}{\partial x \partial y} = 0 \;\wedge\; \frac{\partial^2 h}{\partial x^2} = 0$ whereby $\frac{\partial^2 h}{\partial y^2} \neq 0 \;\vee\; \frac{\partial^2 h}{\partial y^2} = 0,$

2. $\frac{\partial h}{\partial x} \neq 0 \;\wedge\; \frac{\partial h}{\partial y} = 0 \;\Rightarrow\; \frac{\partial^2 h}{\partial y^2} = 0 \;\wedge\; \frac{\partial^2 h}{\partial y \partial x} = 0$ whereby $\frac{\partial^2 h}{\partial x^2} \neq 0 \;\vee\; \frac{\partial^2 h}{\partial x^2} = 0,$

3. $\frac{\partial h}{\partial x} \neq 0 \;\wedge\; \frac{\partial h}{\partial y} \neq 0 \;\Rightarrow\; \frac{\partial^2 h}{\partial x^2} = 0 \;\wedge\; \frac{\partial^2 h}{\partial y \partial x} = 0 \;\wedge\; \frac{\partial^2 h}{\partial y^2} = 0.$

The structural fields of slope or gradient magnitude and aspect have except singular points constrained to the peak, pit and saddle points of original scalar fields also the singular points, where zero isolines of the profile curvature and tangential or plan curvatures are passing, but for which conditions triplets described above are not valid. This means, that these conditions can be sufficient but they do not have to be the necessary conditions.

In the both triplets of described conditions for discriminant D_2 of second fundamental form of the surface the following is valid:

$$D_2 = \left(\frac{\partial^2 h}{\partial x \partial y}\right)^2 - \frac{\partial^2 h}{\partial x^2}\ \frac{\partial^2 h}{\partial y^2} = 0 \ .$$

This simultaneously is the condition for the parabolic points of regular surface. By testing the relevant structural scalar fields we found out, that its singular points, non-constrained to the peak, pit and saddle points of original scalar field, are identical with the intersection points of the zero isolines of the profile curvature and tangential or plan curvatures and zero isolines of the discriminant D_2. Thus the discriminant D_2 is from our point of view an important LSP.

In accordance with paper [8], in the scalar field of slope angle γ_N or gradient magnitude $|\nabla h|$, the isoline $S_t = 0$, which is identical with isoline $D_t = 0$ and passes through the set of points with extreme value of slope angle γ_N or extreme value of the gradient magnitude $|\nabla h|$. Because based on the paper [4] it is valid that $S_t \equiv A_n = 0$, we can supplement the previous sentence to with: "and simultaneously identical with isoline $A_n = 0$". Then, in accordance with this paper, isoline $S_t = D_t = A_n = 0$ and passes through the inflection points of orthogonal projections of space slope curves on the plane of x and y co-ordinates, i.e. on the map and divides these planar curves on the convex and concave parts. LSP of S_t can be interpreted as the planar curvature of the flow lines and in accordance with paper [2], LSP of A_n can be interpreted as the torsion of contour line. The result of above mentioned is as follows: the point (x_o, y_o) is land surface point of change of flow direction if LSPs of S_t , D_t and A_t at the point (x_o, y_o) are equal to zero. In these places of land surface the principal directions of surface (directions of minimal and maximal curvatures) depends from directions of the flow and contour line accordingly.

For isoline of $S_t = D_t = A_n = 0$ it is valid that also in the scalar field of aspect angle A_N it passes through set of points with extreme values of aspect angle A_N . The Fig. 2 illustrates, that isoline $D_2 = 0$ and isoline of $S_t = D_t = A_n = 0$ pass trough singular points of scalar field of slope angle γ_N or gradient magnitude $|\nabla h|$ and through scalar field of aspect angle A_N as well. In the case of the scalar field of slope angle γ_N or gradient magnitude $|\nabla h|$, the following has to be valid for these singular points: $(K_N)_n = S_n = 0$ and in case of scalar field of aspect angle A_N the following has to be valid: $(K_N)_t = K_r = 0$ as well.

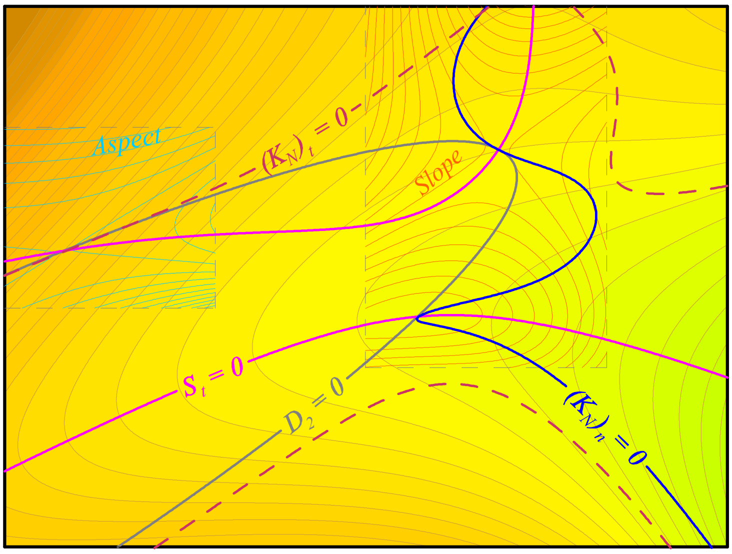

Fig. 2. Intersections the zero isolines of profile $(K_N)_n$ or tangential $(K_N)_t$ curvature and zero isolines of discriminant D_2 of second gauss differential form of surface and zero isolines of LSPs S_t in singular points of aspect and slope isoline fields.

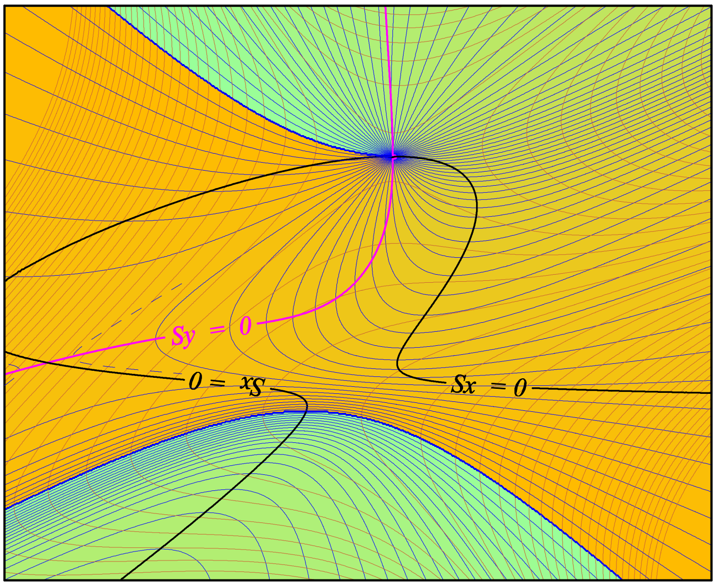

Fig. 3. Isoline field of aspect A_N in the role of the original field and intersections the zero isolines of partial derivatives where $\frac{\partial A_N}{\partial x} = S_x$ and $\frac{\partial A_N}{\partial y} = S_y$.

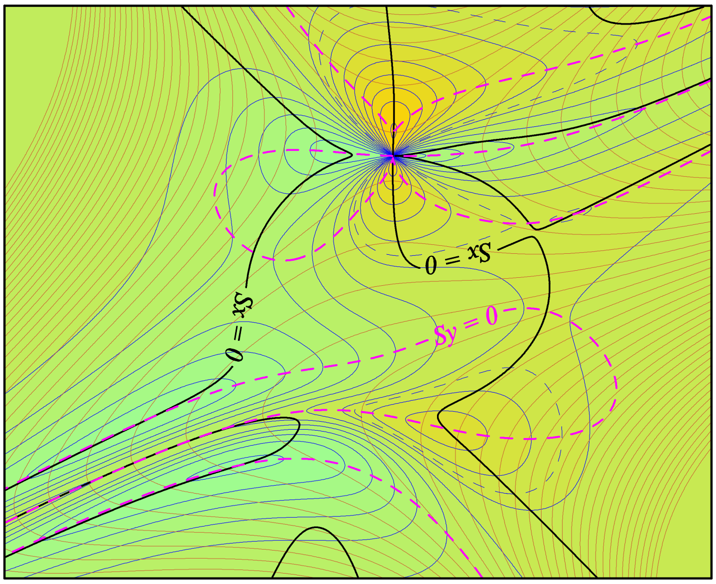

Fig. 4. Isoline field of profile curvature $(K_N)_n$ in the role of the original field and intersections the zero isolines of partial derivatives where $\frac{\partial (K_N)_n}{\partial x} = S_x$ and $\frac{\partial (K_N)_n}{\partial y} = S_y$.

Identification of the singular points in the scalar and its isoline fields of LSPs of second and higher orders isn't such simple problem. In accordance with in the paper mentioned method for derivation of particular LSPs, can, in case of LSPs of second and higher orders, only be formulized the rule, which concerns the occurrence of its extremes. The rule states, that the zero isolines of LSPs pass through the extremes of abstract scalar fields of second and higher order LSPs. The zero isolines of LSP are derived from one-degree lower LSP functions by its derivation in the direction based on which function of LSP was defined and the rule applies only to the function. For example: the isolines

$$(K_N)_{nn} = \frac{\partial (K_N)_n}{\partial n} = \frac{\partial (S_n \cos \gamma_N)}{\partial n} = 0 \, , \, A_{tt} = \frac{\partial A_t}{\partial t} = \frac{\partial^2 A_N}{\partial t^2} = 0 \, , \, (K_N)_{tt} = \frac{\partial (K_N)_t}{\partial t} = \frac{\partial (A_t \sin \gamma_N)}{\partial t} = 0 \, ,$$

$$D_{tt} = \frac{\partial D_t}{\partial t} = \frac{\partial^2 |\nabla h|}{\partial t^2} = 0 \, , \; S_{tt} = \frac{\partial S_t}{\partial t} = \frac{\partial^2 \gamma_N}{\partial t^2} = 0 \, , \; A_{nn} = \frac{\partial A_n}{\partial n} = \frac{\partial^2 A_N}{\partial n^2} = 0$$

pass through the singular points of the scalar fields of the profile curvature, tangential and plan curvatures and D_t, S_t and A_n curvatures respectively.

Each LSP which is the element of set G_{RF} except the altitude h, creates the abstract scalar field. This scalar field is the structural field of original scalar field of altitudes defined by function $h = h\ (x, y)$. We can consider these scalar fields, even if these are the abstract scalar fields, as an original scalar fields such that for the selected LSP, we can consider function $LSP = f_{LSP}\ (x, y)$. This expression is an identical form with equation $s = f_s\ (x, y)$. Then the following has to be valid: $LSP \equiv s \equiv h$ and selected LSP will be the single element of first subset ${}^{(0)}G_{RF}$ of set G_{RF}. If the first order partial derivatives

$$\frac{\partial f_{LSP}}{\partial x} \text{ and } \frac{\partial f_{LSP}}{\partial y}$$

at the point (x_o, y_o) of function $LSP = f_{LSP}\ (x, y)$ are equal to zero, then the point (x_o, y_o) is the singular point of abstract surface of particular LSP. This means, that singular points of particular LSPs can be located based on the intersection points of the zero isolines of the first order partial derivatives of their field functions with respect to x and y variable. These LSPs now have status as an original scalar field of altitudes. There are outlines of zero isolines of the first order partial derivatives

$$\frac{\partial A_N}{\partial x}, \frac{\partial A_N}{\partial y} \text{ and } \frac{\partial (K_N)_n}{\partial x}, \frac{\partial (K_N)_n}{\partial y}$$

of non-continuous function of scalar field of aspect A_N and function of scalar field of profile curvature $(K_N)_n$ in the role of the original scalar fields at the Fig. 3, 4.

4 Conclusions

For singular points of the function of the two variables $f_s\,(x, y)$ following mathematical conditions are valid:

$$\frac{\partial f_s}{\partial x} = \frac{\partial f_s}{\partial y} = 0 \, .$$

To identify the points for which these conditions are valid, we need the tools which are able to generate the partial derivatives of the modeled scalar or isoline fields. These tools are present in GIS basis for DEM, and are capable to deliver not only visualization outputs, but outputs based on the analysis of the DEM. Regrettably, the current GIS and DEM software mostly do not enable us to generate output datafiles of the partial derivatives of the used interpolation functions. It is a pity, because the simplest way for finding of faults in every modeled scalar field is automated detection of its singular points.

Having access to directional derivatives of the modeled scalar field, in the form of output files generated by the DEM tools, allows us to find singularities or to calculate new morphometric land surface parameters without using the complicated programming procedures.

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