Edge and Ridge Detectors as a Techniques for Geomorphometry

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Abstract. Edge and Ridge Detectors as a Techniques for Geomorphometry. This paper describes methods for ridge or valley and edge detection of the digital elevation model. Ridge or valley and edge lines detection is the first step of automated delimitation of elementary forms of georelief. Methods of image processing (Neighbourhood algorithm and Canny edge detector) and analytical methods for edge and ridge detection as a potential tools for landform analysis are compared in this paper.

Keywords: terrain skeleton, ridge lines, valley lines, edge lines, curvatures, land surface parameters

1 Introduction

An elementary morphodynamical form can be considered as a general continuous geometrical surface. Within the borders of such a form are suitable conditions for similar dynamical processes. In the areas where is geometrical and dynamical continuity corrupted is a sudden change in geometrical and dynamical properties of the surface. It is desired to define the boundaries between particular morphodynamical forms of the surface in these points. These boundaries are as well the boundaries of elementary forms of georelief. The change of single process dynamic affecting the georelief does not have to create morphologically strong boundaries. The delimitation of boundaries between morphodynamical forms is therefore not always unambiguous. This results from the fact that it is impossible to delimitate sharp-boundary value intervals of morphometrical characteristics used to define the first morphodynamical form and values of morphometrical characteristics defining the second morphodynamical form.

In some cases is therefore better to define the boundaries of particular morphodynamical forms of georelief by specific lines in the georelief where it comes to change of the whole complex of morphometrical characteristics of georelief. Such lines are for example ridge and valley lines which created the terrain skeleton or notional edge lines on smooth and continuous surface.

We can find the delimitation of boundaries of elementary surfaces in the frame of mathematical objects – manifolds and as well in the digital image processing. Many of the methods applied in the field of computer graphics are based on searching for specific lines of computing image. Detection of edges and ridges became one of the basic techniques for computer visualization of real objects.

2 Detection of ridge and valley lines

According to paper [10], a natural way to construct such a system suitable for edge or ridge detections goes as follows: Let introduce at any point (x_o, y_o) in a two-dimensional image a local coordinate system (u, v) with the v-axis parallel to the gradient direction at point (x_o, y_o) and the u-direction perpendicular. Directional derivatives in this local coordinate system (u, v) are related to partial derivatives in the Cartesian coordinate system (x, y) by

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\sin\alpha - \frac{\partial f}{\partial y}\cos\alpha \ , \tag{1}$$

$$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\cos\alpha + \frac{\partial f}{\partial y}\sin\alpha \ .$$

The angle α in equations (1) can be expressed by the directional cosines of unit normal vector $\overset{\circ}{\boldsymbol{n}}$ oriented in the direction of gradient vector:

$$\cos\alpha = \frac{\mathrm{d}x}{\mathrm{d}\boldsymbol{n}} = \frac{\frac{\partial f}{\partial x}}{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}}, \qquad \cos\beta = \sin\alpha = \frac{\mathrm{d}y}{\mathrm{d}\boldsymbol{n}} = \frac{\frac{\partial f}{\partial y}}{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}}. \tag{2}$$

Local coordinate system (u, v) is characterized by the fact that first order partial derivative $\partial f/\partial u$ is equal to zero [9].

We arise from the idea that ridge or valley lines of a function with n-variables are composed of set of curves, whose points are the local minims or local maxims in the n-1 dimension. Ridge and valley lines of smooth and continuous function of two variables in the general form $z = f_z\ (x, y)$ are curves for whose points holds that they are local minims and maxims in the direction of axis u of local coordinate system (u, v) perpendicular to the gradient vector. This means, that local coordinate system (u, v) at the point (x_o, y_o) of a surface corresponds with its principal directions, so for the partial derivative of the second order is valid

$$\frac{\partial^2 f}{\partial u \partial v} = 0. \tag{3}$$

Not all parts of the curves, for which holds the previous condition, have the character of ridge or valley line. This means that this condition is not sufficient for definition of ridge and valley lines. This condition is in case of non-degenerate function $z = f_z\ (x, y)$ necessary to replenish (according to paper [9] or [10]) with condition

$$\left(\frac{\partial^2 f}{\partial u^2}\right)^2 - \left(\frac{\partial^2 f}{\partial v^2}\right)^2 = \left(\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2\right)\left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right) - 4\,\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\,\frac{\partial^2 f}{\partial x \partial y} > 0, \tag{4}$$

which arise from the assumption, that the absolute value of the contour tangent change of gradient is always higher then the absolute value of normal change of gradient. If the selected point is a part of valley line, there must be fulfilled the additional condition

$$\frac{\partial^2 f}{\partial u^2} > 0 \ , \tag{5}$$

i. e. the contour line passing through this point must be convex. For point which is part of a ridge line must be hold the opposite condition

$$\frac{\partial^2 f}{\partial u^2} < 0 \ , \tag{6}$$

i. e. the contour line passing through this point must be concave.

By definition we may consider georelief sites where the condition (4) is not convenient. To specify the ridge and valley lines we have to take into account another condition. Such a condition may be for instance the condition that contour line is perpendicular to ridge or valley line – similar to a condition proposed by Márkus in paper [11]. The character of this condition has the following equation

$$\frac{\partial f}{\partial x} \frac{\partial\left(\frac{\partial^2 f}{\partial u \partial v} = 0\right)}{\partial x} + \frac{\partial f}{\partial y} \frac{\partial\left(\frac{\partial^2 f}{\partial u \partial v} = 0\right)}{\partial y} = 0 \, . \tag{7}$$

For partial derivatives of the second order in the local coordinate system (u, v) mentioned in the previous conditions and for Cartesian partial derivatives in the coordinate system (x, y) hold the following

$$\left(\frac{\partial f}{\partial v}\right)^2 \frac{\partial^2 f}{\partial u^2} = \left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial y^2} - 2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x^2} \ , \tag{8}$$

$$\left(\frac{\partial f}{\partial v}\right)^2 \frac{\partial^2 f}{\partial u \partial v} = \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right) + \frac{\partial^2 f}{\partial x \partial y} \left(\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2\right),$$

$$\left(\frac{\partial f}{\partial v}\right)^2 \frac{\partial^2 f}{\partial v^2} = \left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial y^2} \ .$$

If we modify the second equation in (8) along with relation (3) we receive a relation for expressing the zero isolines of trinity land surface parameters (LSPs). The first LSP - tangent change of gradient is introduced in paper [12] and in [8] denoted as D_t, the second - tangent change of slope and the third - normal change of slope aspect denoted in [3] as S_t and A_n [1].

[1] Whereby

$$D_t = \frac{\frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right) + \frac{\partial^2 f}{\partial x \partial y} \left(\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2\right)}{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}, \qquad S_t = \frac{\frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right) + \frac{\partial^2 f}{\partial x \partial y} \left(\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2\right)}{\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)\left(1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)},$$

$$A_n = \frac{\frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right) + \frac{\partial^2 f}{\partial x \partial y} \left(\left(\frac{\partial f}{\partial y}\right)^2 - \left(\frac{\partial f}{\partial x}\right)^2\right)}{\sqrt{\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)^3}}.$$

LSP of A_n introduced by Sharry in paper [16] (called rotor). Rotor is the curvature of flow lines (slope lines) on a topographic map. It characterizes flow lines twisting [17].

According to the paper [8], in the scalar field of slope denoted as γ_N or the absolute value of gradient (it's magnitude) denoted as $|\boldsymbol{grad}\, z|$ or $|\nabla h|$ isoline S_t = 0. Isoline S_t = 0 is identical with the isoline D_t = 0, passes trough the set of points with the extreme values of slope γ_N or extreme absolute values of gradient $|\boldsymbol{grad}\, z|$. According to paper [3] hold that $S_t \equiv A_n = 0$, we can replenish the previous sentence with „and is identical with the isoline $A_n = 0$". If the LSPs of S_t, D_t and A_n in point (x_o, y_o) are equal to the zero, than according to paper [4] is the point (x_o, y_o) a point on the georelief in which is changing the flow direction (inflexion point of the slope line in Cartesian plain (x, y)) and the principal directions of the surface are in this point of georelief dependent on the direction of flow and contour line. Based on this preposition and prepositions from the previous subsections, we can write the condition (7) as follows

$$\frac{\partial f}{\partial x}\,\frac{\partial(D_t = 0 \vee S_t = 0 \vee A_n = 0)}{\partial x} + \frac{\partial f}{\partial y}\,\frac{\partial(D_t = 0 \vee S_t = 0 \vee A_n = 0)}{\partial y} = 0. \tag{9}$$

We have to note that the second members of the first equation in (8) is identical with numerator in relations describing the two following LSPs. The first LSP from the couple expressing the contour curvature, also called a plan curvature, the second one expresses normal curvature in the tangent direction $(K_N)_t$, called a tangential curvature[2]. The opposite oriented plan curvature, called a horizontal curvature was expressed as a LSP for the first time in paper [5] and denoted as K_r [2]. The opposite oriented normal curvature $(K_N)_t$ was introduced as a LSP for the first time in paper [6] and denoted as K_H. By horizontal and tangential curvature we can substitute the condition (5) and (6) by the following two conditions

$$-K_r \vee (K_N)_t > 0 \;\; ; \;\; -K_r \vee (K_N)_t < 0 \,. \tag{10}$$

3 Edge detection

Edge is a point at which the digital image brightness changes sharply. We may define edge as a point at which the gradient magnitude assumes a maximum in the gradient direction [1]. It is an inflexion point of the slope curve. In the local coordinate system (u, v), for such points hold the following conditions

$$\frac{\partial^2 f}{\partial v^2} = 0\,, \tag{11}$$

[2] Whereby

$$(K_N)_t = \frac{\left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial y^2} - 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x^2}}{\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)\sqrt{\left(1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)}}\;,\quad K_r = -\frac{\left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial y^2} - 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x^2}}{\sqrt{\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)^3}}.$$

$$\frac{\partial^3 f}{\partial v^3} < 0 \,. \tag{12}$$

If we modify the third equation from (8) according to relation (11) we get the relation for expressing the zero isolines of other known LSPs. Their importance for geomorphometry was sketched in paper [2]. The first LSP expresses normal change of gradient. It was derived (with opposite sign) in paper [7] and in paper [8] denoted as D_n or in paper [13] denoted as G_n [3]. The second LSP – profile curvature represents the normal curvature of georelief in the direction of gradient vector $(K_N)_n$ introduced as a LSP for the first time (with opposite sign) in paper [5] and denoted as ω [3]. The third LSP – normal change of slope was derived (with opposite sign) in paper [3] and denoted as S_n [3]. Zero isolines of partial derivatives of the second and third order used in relations (11) and (12) should pass the zero isolines of LSP of ω_n from paper [3] or LSP of D_{nn} from paper [8] or LSP of S_{nn} from paper [3] or [8]. These intersection points are intersection points of zero isolines of both partial derivatives as well. LSPs of D_{nn}, ω_n and S_{nn} was derived by derivation of morphometrical functions $\boldsymbol{D_n}$, ω and S_n in the normal direction. We can substitute the condition (12) by

$$D_{nn} \vee \omega_n \vee S_{nn} < 0 \,. \tag{13}$$

Unlike the methods of computer graphic in the case of modelling the compound slopes are from the geomorphologic point of view interesting all inflexion points laying on the set of slope curve. That means also the inflexion points at which are the minimums of gradient magnitude. For such inflexion points of slope curves holds instead of condition (12) following condition

$$\frac{\partial^3 f}{\partial v^3} > 0 \,. \tag{14}$$

4. Comparison of ridge and valley lines and edges acquired by different methods

The development of ridge and valley lines and edges computed by presented equations (fig. 1) was compared in the frame of testing area with identical lines from fig. 2 delimitated by the digital image edge detectors applied on the same testing area in paper [14]. Compared ridge and valley lines were in paper [14] delimitated by detecting local extremes with the help of the Canny edge detector. The process of searching for local extremes consists of two steps.

[3] Whereby

$$\boldsymbol{D_n} \equiv \boldsymbol{G_n} = \frac{\left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial x^2} + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial y^2}}{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}, \quad S_n = -\frac{\left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial x^2} + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial y^2}}{\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)\left(1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)},$$

$$(K_N)_{\boldsymbol{n}} \equiv -\omega = \frac{\left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial x^2} + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} + \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial y^2}}{\left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)\sqrt{\left(1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right)^3}} \,.$$

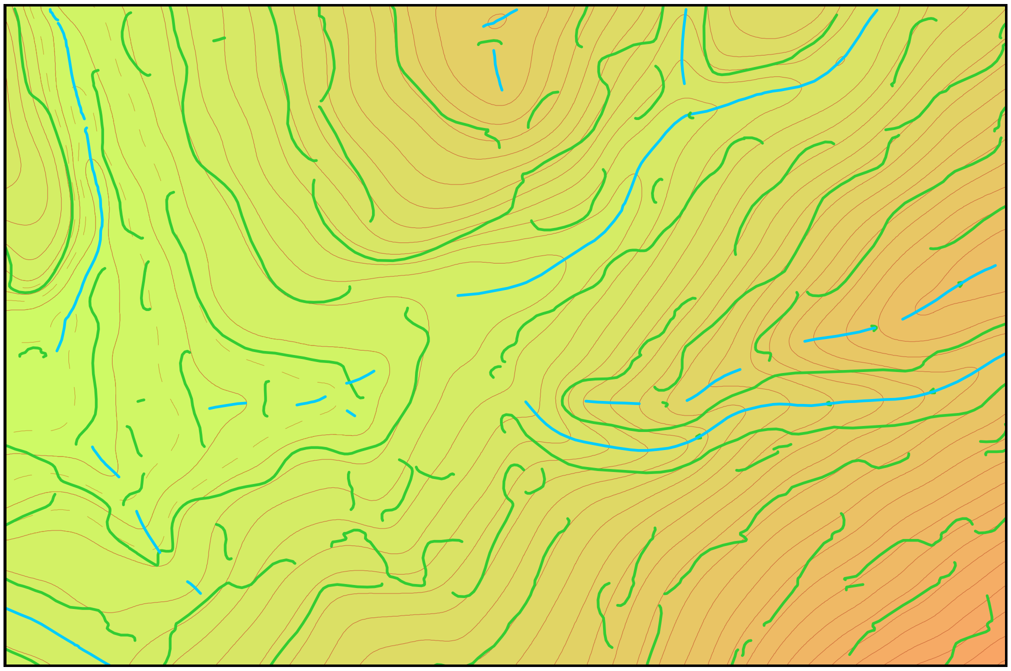

Fig. 1. Valley or ridge and edge lines deliminated by function analysis:
blue colour line - valley or ridge
green colour line - edge

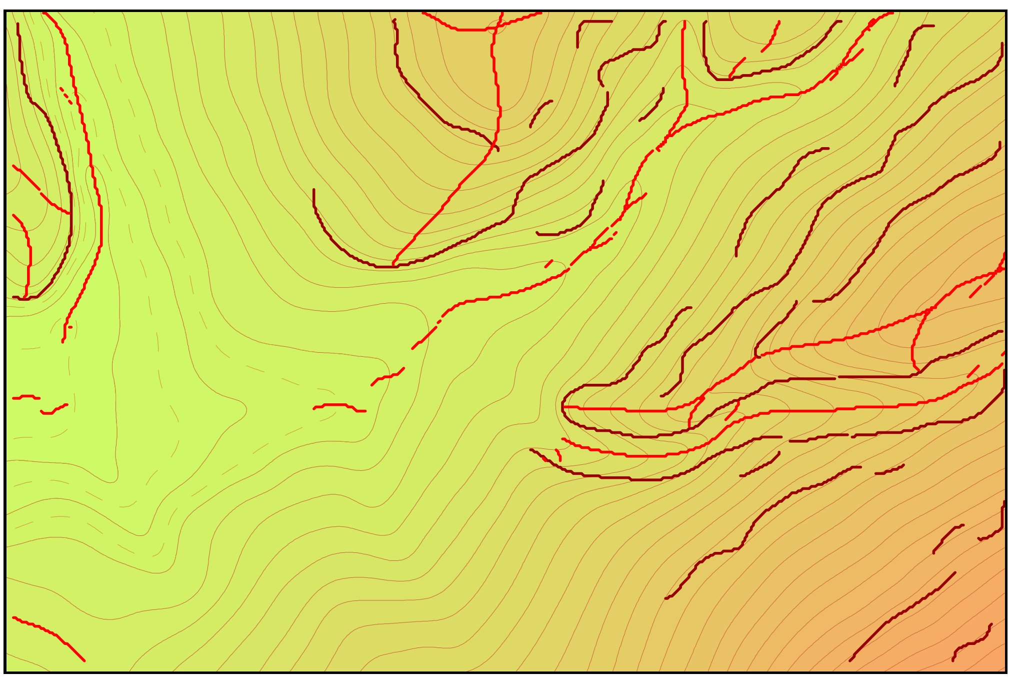

Fig. 2. Valley or ridge and edge lines deliminated by Canny edge detector:
red colour line - valley or ridge
nigger brown colour line - edge

In the first step was applied the Canny edge detector. The results are edges corresponding to inflexion points of slope curves of the altitude field with maximum values of slope. In the next step are in-between these inflexion points found the local altitude maxims and minims by testing the cell neighbourhoods.

The development of edges and ridge and valley lines delimitated by the both above described methods on fig. 3 and 4 is shown. The usage of Canny edge detector is followed by application of noise eliminating filters in the input data and tresholding the gradient to eliminate weak edges. That is the reason why are on fig. 3 lines delimitated strictly from relevant values of derivatives from (11) and (12) with higher density. But in the locations where the edges were delimitated by the Canny edge detector – the edges have the same development as lines delimitated from the derivatives values. From this results that the Canny edge detector is based on gradient computation as well.

In the case of ridge and valley lines we may consider (based on fig. 4) that by applying of the both methods we acquire almost the same range of ridge and valley lines, but in the case of lines delimitated by the Canny edge detector with a little lower accuracy.

In the both cases we can not avoid the problem with evident absence of ridge and valley lines in the area with featureless contour curvature. Here we need to say, that the method for searching of local extremes of contour curvature (sketched for the first time in [11]) is not producing better results as the two methods presented in this paper. In the case of this method may be sufficient the following two conditions

$$A_{tt} = 0 \,, \tag{15}$$

$$\frac{\partial f}{\partial x}\frac{\partial (A_{tt}=0)}{\partial x} + \frac{\partial f}{\partial y}\frac{\partial (A_{tt}=0)}{\partial y} = 0 \,, \tag{16}$$

where LSP of A_{tt} is expressing the tangent change of horizontal curvature[4]. It was introduced in paper [3]. The results of this method are influenced by the requirement of partial derivatives not only of the third order, by of the fourth order and there are huge inaccuracies coming from derivation of such high order. By the usage of this method, we are additionally facing the problem of creating false ridge and valley lines (fig. 5).

[4] Whereby if

$$B = 2\frac{\partial f}{\partial x}\left(\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} + \frac{\partial^3 f}{\partial x^2 \partial y}\frac{\partial f}{\partial y}\right) - \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^3 f}{\partial x^3} - \left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^3 f}{\partial y^2 \partial x}, \quad M = \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial x}\right)^2,$$

$$C = 2\frac{\partial f}{\partial y}\left(\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} + \frac{\partial^3 f}{\partial y^2 \partial x}\frac{\partial f}{\partial x}\right) - \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^3 f}{\partial x^2 y} - \left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^3 f}{\partial y^3}, \quad N = \frac{\partial^2 f}{\partial x^2}\frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial f}{\partial y},$$

$$K = 2\frac{\partial^2 f}{\partial x \partial y}\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} - \left(\frac{\partial f}{\partial y}\right)^2 \frac{\partial^2 f}{\partial x^2} - \left(\frac{\partial f}{\partial x}\right)^2 \frac{\partial^2 f}{\partial y^2}, \quad O = \frac{\partial^2 f}{\partial y^2}\frac{\partial f}{\partial y} + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial f}{\partial x},$$

then

$$A_{tt} = \frac{\frac{\partial f}{\partial y}\,(BM - 3\,KN) - \frac{\partial f}{\partial x}\,(CM - 3\,KO)}{M^3}.$$

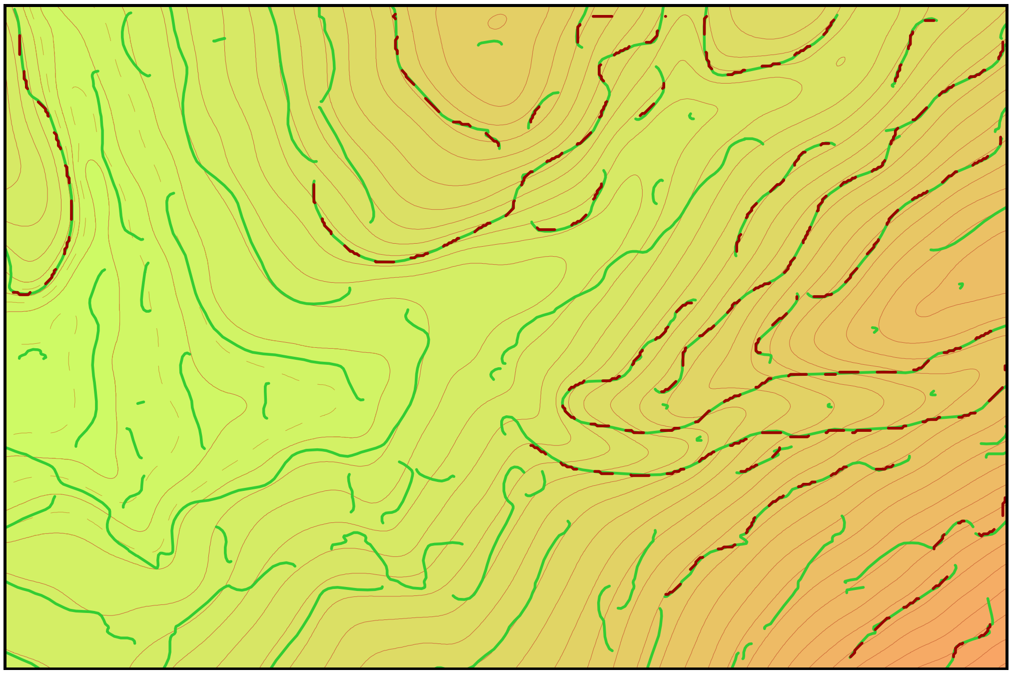

Fig. 3. Edges deliminated by function analysis and Canny edge detector:
green colour line - edges deliminated by function analysis
dashed brown colour line - edges deliminated by Canny edge detector

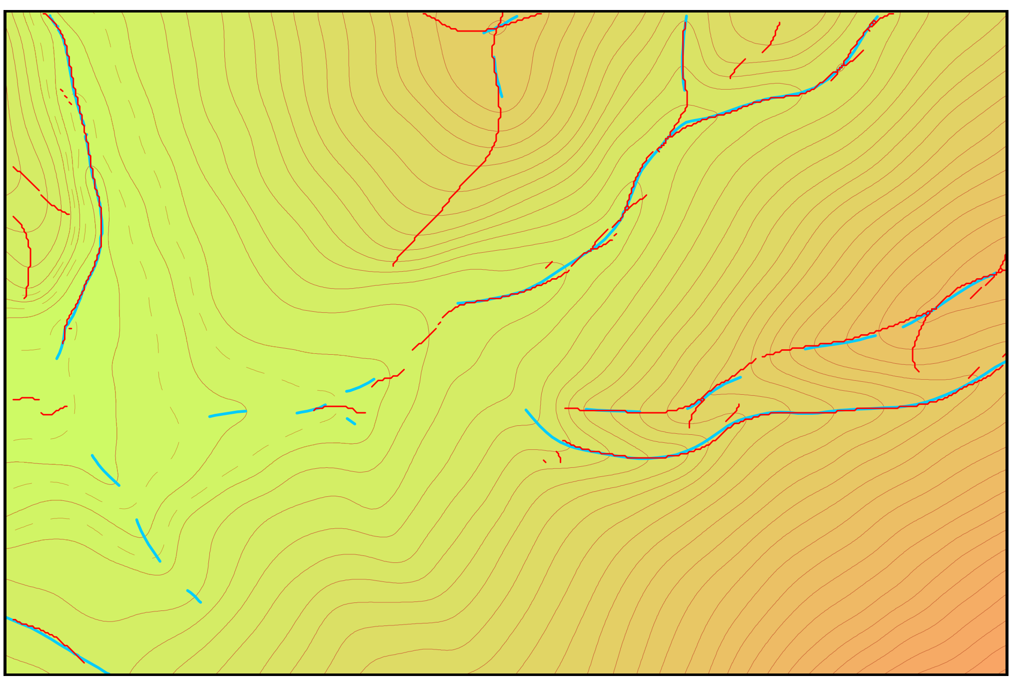

Fig. 4. Valley and ridge lines deliminated by function analysis and Canny edge detector:
blue colour line - valley and ridge lines deliminated by function analysis
thin red colour line - valley and ridge lines deliminated by Canny edge detector

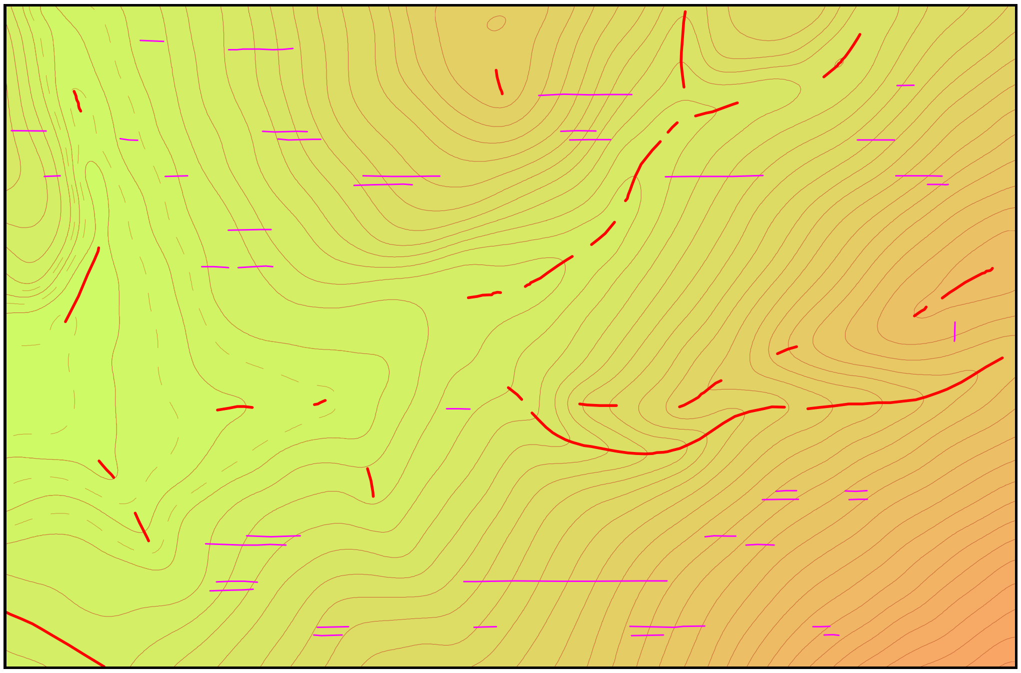

Fig. 5. Valley and ridge lines deliminated by terms (15) and (16):
red colour line - valley and ridge lines
thin purple colour line - false valley and ridge lines

The development of ridge and valley lines on presented figures (deliminated by function analysis) is conditioned by the allowed deviation in the development of zero isolines of the calculated couples of relevant LSPs. This deviation exceeds not half thickness of false ridge and valley line on the fig. 5. The method for approximation of partial derivatives of higher orders is described in paper [15].

5 Conclusion

Ridge and valley lines and inflexion points of slope curves are locations with large change of morphometrical characteristic of any surface. We do not avoid its detection even in digital image processing techniques. These techniques can be applied for georelief analysis as well. The theoretical background of methods used for delimitation of ridge and valley lines, for digital image processing and as well for georelief analysis are very close and lead to similar results. The proof of this is the comparison of methods for ridge and valley lines detection.

These methods are producing good results in the case of rugged topography, but even in this case we can not avoid the problem of identification of discontinuous lines. By removing the discontinuities in the ridge and valley lines, we probably wouldn't avoid the application of pure graphical methods. It implies from the character of the curves, which is very difficult to describe by one ore more formalized rules.

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