# Insolation on the Steep North Slopes and Model r.sun

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**Abstract.** Sun position above the tangent plane to the land surface is always equal to sun position above the horizon at the point of observation with another geographical coordinates. If we determine these coordinates, we can calculate the solar insolation at the land surface by a simple mathematical model. This method is applied in solar insolation model r.sun, prepared for an open source environment of GRASS GIS. Because is GRASS GIS freeware, model r.sun is available for many of users. Therefore is useful to test the quality of r.sun model.

**Keywords:** direct insolation, solar incidence angle, solar hour angle, solar radiation model

## 1 Introduction

Intensity of solar radiation at the land surface depends not only on season, geographical position and physical conditions of atmosphere but on the land surface forms. It is a very significant in the north middle latitudes at the hill slopes with northern aspect and high slope angle. Sunbeams do not fall directly on these slopes during some winter days. Sun is shining on them twice during some summer days. This results from the fact, that the sun position above the tangent plane to the land surface at the given point is always equal to sun position above the parallel tangent plane to the Earth's surface[3] at the point with unknown geographic position. Morphometric variables of the land surface - aspect *A* and slope angle *S* at given point are defining this position exactly by a new geographical coordinates. If we determine these coordinates, we can calculate the characteristics of direct insolation (solar incidence angle, insolation time, etc.) by a simple mathematical model (solar elevation equation (2)).

## 2 Data and methods

For solar azimuths $\alpha_r$ and $\alpha_s$ during sunrise and sunset (angle $h_H$ - solar elevation above the horizon is equal to zero) at a point on the Earth's surface at latitude $\varphi$, following equation is valid:

$$\mathrm{tg}\ \alpha_{r,s} = \frac{C_{22}\sqrt{1-\frac{C_{33}^2}{C_{31}^2}}}{\frac{C_{33}\,C_{11}}{C_{31}} + C_{13}} \qquad \textbf{(1)}$$

which, for example, we can found in [8], whereby

$$\begin{array}{ll} C_{11} = \sin\varphi\cos\delta & C_{13} = -\cos\varphi\sin\delta \\ C_{22} = \cos\delta & \\ C_{31} = \cos\varphi\cos\delta & C_{33} = \sin\varphi\sin\delta \end{array} \qquad \textbf{(1a)}$$

and $\delta$ is the solar declination[4].

[3] Surface of the Earth`s sfere

[4] The declination $\delta$ during any given day can be considered constant value in engineering calculations.

Solar azimuth $\alpha_{r,s}$ in latitudes above the Tropic of Cancer, from spring equinox to autumnal equinox, is northeast during the sunrise and northwest during the sunset. It means that the north facing surface in northern temperate zone can be direct insolated twice a day – at firstly time after the sunrise and at second before the sunset. It is possible to see it in flats with rooms oriented to the north. Sun is shining to the windows in the morning, before noon it shines only to the roof and to the windows again in the afternoon.

This fact can be easily overlooked. From the results of the quadratic hour angles equations in papers [1, 2, 8] for hour angle at the time when a tilted surface is *just* irradiated by the direct insolation and *just* shaded from the direct insolation it is not explicitly clear, which solution defines first hour angle and which second. Interpretation is simple if we use non quadratic equation based on the following assumption: "Aspect angle $A$ and slope angle $S$ at the land surface point with latitude $\varphi$ and longitude $\lambda$ is defining latitude $\varphi'$ and relative longitude $\lambda'$ with respect to longitude $\lambda$. Tangent plane to the Earth's surface at the point with latitude $\varphi'$ and longitude $\lambda + \lambda'$ is parallel with tangent plane to the land surface at the point with latitude $\varphi$ and longitude $\lambda$. The planes are with the same angle to the sunbeams". We call the latitude $\varphi'$ and relative longitude $\lambda'$ as a *reference* coordinates.

Let us to define tangent plane to the land surface at a point with coordinates ($\varphi$, $\lambda$). Tangent plane is not parallel with plane of local horizon if slope angle $S > 0°$. Relation of its plane to the plane of the local horizon is defined by vertical slope angle $S$ and horizontal aspect angle $A$. We start from a known solar elevation equation:

$$\sin h_H = C_{31} \cos \omega + C_{33}\,, \qquad (\mathbf{2})$$

$\omega$ is the hour angle[5] and parameters $C_{31}$ and $C_{33}$ , are from Equations (1a). Angle $h_{LS}$ between the sunbeam and tangent plane to the land surface at point with latitude $\varphi$ and longitude $\lambda$ at selected hour angle $\omega$ is then defined by equation:

$$\sin h_{LS} = C'_{31} \cos(\omega - \lambda') + C'_{33}\,. \qquad (\mathbf{3})$$

Following formulas are valid for parameters $C'_{31}$ a $C'_{33}$ :

$$C'_{31} = \cos\varphi' \cos\delta \qquad (\mathbf{3a})$$
$$C'_{33} = \sin\varphi' \sin\delta$$

If aspect angle $A$ is measured eastward from the south for reference coordinates ($\varphi'$, $\lambda'$) following relations are valid:

$$\sin\varphi' = -\cos\varphi \sin S \cos A + \sin\varphi \cos S\,, \qquad (\mathbf{4})$$

$$\operatorname{tg}\lambda' = \frac{-\sin S \sin A}{\sin\varphi \sin S \cos A + \cos\varphi \cos S} \qquad (\mathbf{5})$$

[6].

Hour angle $\omega_{hmax}$ at maximum value of angle $h_{LS}$ at tested point is defined by equality $\omega_{hmax} = \lambda'$.

Angle $h_{LS}$ is a solar incidence angle. If solar incidence angle $h_{LS}$ is zero we can transform the Equation (3) into the form:

$$\cos\left((\omega_{r,s})_{LS} - \lambda'\right) = -\frac{C'_{33}}{C'_{31}}\,, \qquad (\mathbf{6})$$

$(\omega_r)_{LS}$ and $(\omega_r)_{LS}$ are hour angles at the times when a tilted surface is irradiated by the direct insolation, and shaded from the direct insolation. It is relative "sunrise" and "sunset" (above/over the tangent plane to the land surface). Given period of potential daily insolation time[6] we get after the modification of hour angles of $(\omega_r)_{LS}$ and $(\omega_s)_{LS}$ with respect to the solar hour angle at true sunrise and

[5] Equation $\omega = \pi(t - 12)/12$ is a relation between hour angle $\omega$ in radians measured westward from the observer's meridian and solar time $t$ .

[6] In the case of clear sky, the time when direct insolation approaches the land surface.

sunset (above/over the horizon). Solar hour angle at true sunrise $(\omega_r)_H$ and true sunset $(\omega_s)_H$ is defined by following equation[7]

$$\cos\,(\omega_{r,s})_H = -\frac{C_{33}}{C_{31}}\,. \qquad \textbf{(7)}$$

Equation (3) is analogical with solar incidence angle equations from different authors [3, 5, 7, 8]. Main interpretation advantage of equation (3) is the calculation of the reference coordinates ($\varphi'$, $\lambda'$) of point on the Earth's surface with the same incidence of sunbeams.
So for example, theoretically is possible that points located at 49 degrees north latitude on north-oriented slopes with slope angle $S$ greater than 64.44° = (90° - 49° + 23.44°) are insolated twice a day in period from spring equinox day to autumnal equinox day and in period from autumnal equinox day to spring equinox day they are not insolated[8]. With coming of midsummer day will rise the deviation of orientation from the north orientation ($A$ = 180°). During midsummer day, points on the land surface with slope angle values higher than 64.44° can be insolated twice a day if value of aspect angle ranged from 142.7° to 217.3°.
In this cases a reference coordinates $\varphi'$ and $\lambda'$ are determining points, which are located south from the Arctic polar circle on the northern hemisphere, but on its reverse side. Thereby, the defined position of sun above true horizon at tested land surface points.
In the case of tested twice insolated point with coordinates ($\varphi$, $\lambda$) is observed motion of the sun above the horizon at the point with coordinates ($\varphi'$, $\lambda+\lambda'$) overlapping in time (in its afternoon decreasing phase until sunset) with time after the sunrise above local horizon at tested point. Simultaneously is the observed motion of the sun above local horizon at the point with latitude $\varphi'$ and longitude $\lambda+\lambda'$ overlapping in time (in its morning increasing phase after sunrise) with time before sunset at the tested point. For example, if $(\omega_s)_{LS}$ is negative definite and $(\omega_s)_{LS} > (\omega_r)_H$, than the hour angle $(\omega_s)_{LS}$ determinates the first time a day when tilted surface is shaded from the direct insolation. If also $(\omega_r)_{LS}$ is positive definite and $(\omega_r)_{LS} < (\omega_s)_H$, than the hour angle $(\omega_r)_{LS}$ determinates the second time a day when tilted surface is irradiated by the direct insolation.

## 3 Results and discusion

The described method is applied in solar irradiance and irradiation model r.sun, prepared for an open source environment of GRASS GIS. The position of the sun with respect to an inclined surface in model r.sun is defined by the solar incidence angle equation (3) [9]. According to manual [4] currently, there are two modes of r.sun. In the first mode model computes solar incidence angle or relevant irradiance using the set local time. In the second mode daily sums of insolation time and beam, diffuse or ground reflected irradiation are computed by integrating insolation or relevant irradiance between sunrise and sunset. The user can set a finer or coarser time step.
The negative definite incidence angles are written to the output data set as zero values. Times when tested cell of grid is irradiated by the direct insolation and shaded from the direct insolation between sunrise and sunset are not input data for beam and diffuse irradiation or duration of insolation computation. Solar incidence angle as output from r.sun is therefore important and is useful to follow to test the quality of model r.sun.
Difference between midsummer day potential insolation time computed without the shadowing effect of terrain by model r.sun and by modified model with respect of Equation (6) is showed on the map (Fig. 2). Testing virtual hill is located at 49 degrees north latitude. Sharp boundary created two compact areas. Grid cells with the unacceptable deviation value covered almost all north-east, north and north-west slope. Solar incidence angle at noon as output from r.sun is showed on the Fig. 3. There is an indication of the existence of a discontinuity in the data. Solar incidence angle for a midsummer day at 9 a.m. and at 3 p.m. as output from r.sun is showed on the Fig. 4. There is apparent discontinuity line. Discontinuity begins for strictly north aspect since 41° slope angle. This slope angle value is solution of equation

$$S = 90^\circ - \varphi_{=49^\circ}\,. \qquad \textbf{(8)}$$

[7] Solar elevation $h_H$ from equation (2) is zero.
[8] 23.44° - approximate latitude of the Tropic of Cancer.

That means the assumed deviation can be generated by tangent function (Equation (5)) implemented in a computer program, when relative longitude $\lambda'$ higher as 90° and lesser as -90°, i.e., tangent point of the tangent plane to the Earth's surface with the same incidence of sun's rays as tested point is located on the other side of the northern hemisphere. Domain for tangent function is $-\pi/2 \leq x \leq \pi/2$ in radians, but relative longitude $\lambda'$ is $-\pi \leq \lambda' \leq \pi$ (full circle). Sign of the numerator and denominator of Equation (5) modifies longitude $\lambda'$ in this range.
Used range for relative longitude $\lambda'$ from $-\pi/2$ to $\pi/2$ radians in model r.sun is the reason why only large positive definite solar incidence angles $h_{LS}$ from equation (3) are constrained to the steep north slopes on the Fig. 3. The result is daily sums of beam irradiation in Wh m$^{-2}$ computed by model r.sun without the shadowing effect of terrain on the steep north slopes higher as daily sums of beam irradiation on the steep south slopes[9] (Fig. 5). Then the differences between potential daily sums of beam irradiation on the steep north slopes computed by model r.sun and potential daily sums of beam irradiation computed by modified model with respect range for relative longitude $\lambda'$ from $-\pi$ to $\pi$ radians are very significant, more then tenths of percents.

a) b)

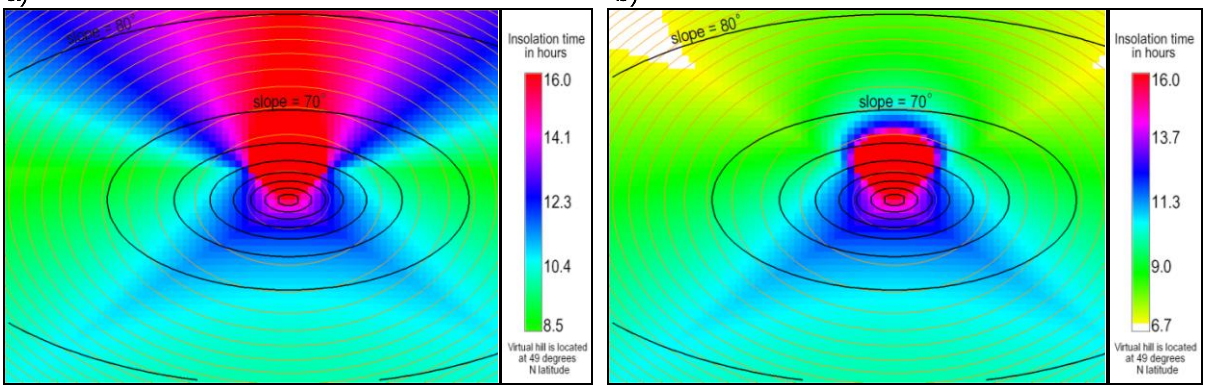


**Fig. 1** Insolation time maps (midsummer day):
a) insolation time is computed by model r.sun without the shadowing effect of terrain
b) insolation time is computed by modified model with respect range for relative longitude $\lambda'$ from $-\pi$ to $\pi$ radians.

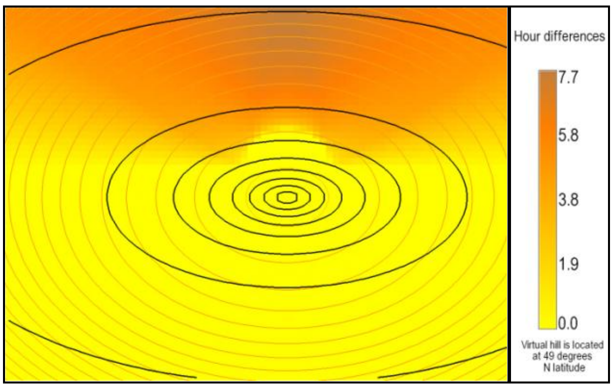


**Fig. 2** Absolute hour difference between potential daily insolations on the maps in Fig. 1.

[9] The beam irradiance on an inclined surface $B_{ic}$ in W m$^{-2}$ is calculated as: $B_{ic} = B_{0c} \sin h_{LS}$ where $B_{0c}$ is the beam irradiance normal to the solar beam and attenuated by the cloudless atmosphere [9].

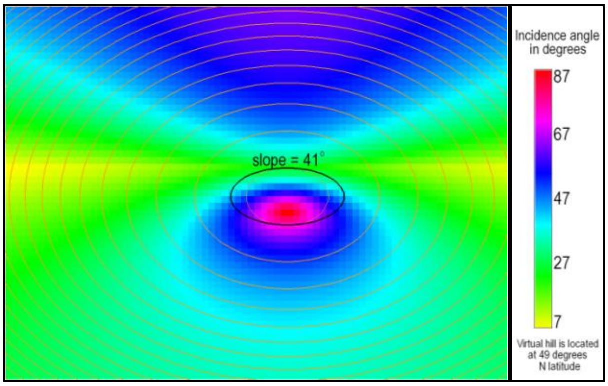


**Fig. 3** Solar incidence angle map (midsummer day at noon):
incidence angle is computed by model r.sun without the shadowing effect of terrain.

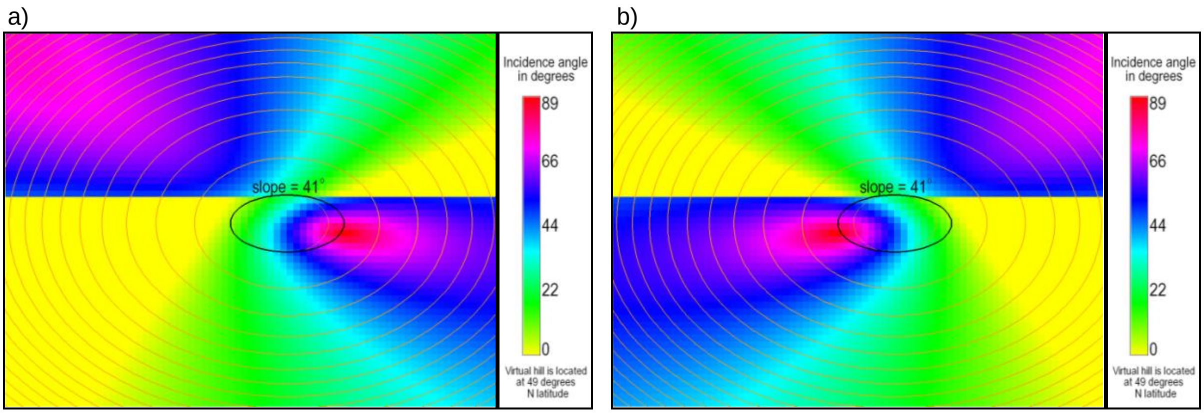


**Fig. 4** Solar incidence angle maps (midsummer day):
a) at 9 a.m.
b) at 3 p.m.
- incidence angle is computed by model r.sun without the shadowing effect of terrain.

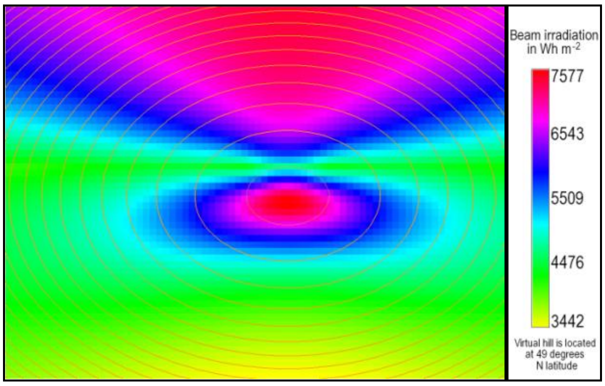


**Fig. 5** Beam irradiation (midsummer day):
beam irradiation is computed by model r.sun without the shadowing effect of terrain.

First raster maps from Fig. 1 and raster maps from Fig. 3, Fig. 4 and Fig. 5 are output of the r.sun module in GRASS GIS version 6.3.0 for MS Windows.

## 4 Conclusions

Modules of topographic solar radiation models for GIS must, for time when a land surface is irradiated by the direct insolation and shaded from the direct insolation, consider with two pairs of values also in the case, when tested point is not shadowed by terrain. This is significant for computation of solar insolation on the steep north slopes. When this condition is not fulfilled, solar radiation model can generate wrong outputs.

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