Interpolation of Rainfall in the Mountainous Region

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Abstract. Knowledge of mountain´s climate variability is still limited, mostly due to a combination of data scarcity and heterogenous orography. The spatial pattern of precipitation is known to be highly depended on meteorological conditions and relief. Nonetheless, the total amount and spatial distribution of mountain precipitation are critical inputs for a variety of a ecological, agricultural and hydrological processes, and are required for estimating rainfall trends in space and time. An accurate assessment of the spatial and temporal variability of precipitation in mountains is therefote a prerequisite for river basin management, given the impact of mountain precipitation on the hydrological cycle in general, and floods, droughts and landslides in particular. The development of high-resolution numerical weather prediction models, and the threat of global climate change, has further increased motivation to better observe the distribution of precipitation and understand the underlying process. The purpose of the investigation was to find a method of mapping rainfall in mountainous region in Jeseniky mountains. Three interpolation methods were discussed – ordinary kriging and simple kriging, inverse distance weighting. The kriging methods require an understanding of the principles of spatial statistics and provide statistically unbiased estimates of surface values from a set of control points.

Keywords: precipitation, interpolation, orographic effect

1 Introduction

Searching and collecting meteorological information are major tasks associated with planning and managing regional water resources development. Rainfall data are frequently used in hydrological analysis. Insufficient or incorrect rainfall observations may cause erroneous hydrological modeling and, hence, affect related hydrological planning and design. Hydrologist often have no choice but to use a rough evaluation when applying other hydrological aspects due to the need for hydrological data on ungauged sites or other missing observations (Cheng, 2007).
Mountainous areas, with their complex relief, are characterized by complex precipitation patterns. Phenomena like the enhancement of precipitation on windward sides (orographic effect) or the rain shadow effect are typical of regions with varied topography. The raingauge networks are typically sparse and of uneven density, covering mainly valley and lowland areas: the information available is inevitably biased towards the lower elevations, and the rainfall patterns are therefore difficult to model. (Prudhomme, 1999).
Orographic influence occurs only in the proximity of high ground in the case of a stable atmosphere. There are three main mountain effect: orographic lifting, thermal forcing and obstacle effects, which include mountain blocking, flow deflection and the production of lee-side flow disturbances. Mountains have two major roles in forming clouds and precipitation. First, mountain are obstacles to atmospheric flow. As stable approaching airflow encounters a mountain barrier, it is forced to ascend the mountain (Banta, 1990). If the necessary atmospheric conditions exist, particularly the stability and moisture content of the atmosphere, clouds and precipitation may be produces as a result of the forced-orographic uplift of the air. To a large extent, the stability of the atmosphere determines how the obstacle will affect the flow. In particular, the stability determines the maximum orographic lifting realizes by the air. Mountains also caused blocking of stable flow. As stable flow approaches the mountain, it slows down owing to the blocking effect.
The second major role that mountains have in forming clouds and precipitation is that they are elevated heat sources (Orville 1965, Banta 1990). The slopes of the mountain protrude into the atmosphere and air next to the mountain slopes is warmed by the heated slopes. Air next to the mountain becomes warmer than that at the same level away from the mountain and over the plains. This procedures a relatively low pressure in the air next to the mountain. The low pressure induces convergence of flow towards the mountain. Air begins to flow up the slope. Provided there is sufficient heating and low stability, the airflow will eventually rise over the peak and break away from the slope. Updrafts then form over the mountain. (Naum, 2007)

Among a large number of interpolation algorithms, geostatistical methods are widely used. Geostatistical theory is based on a stochastic model which allows the derivation of optimal predictions at arbitrary points in the considered region. At the same time, the associated accuracy can be computed. However, the analysis of accuracy is usually restricted to the interpolation phase. Information about accuracy inherent in subsequently drawn maps is often completely disregarded. This can cause overconfidence in the information provided by a map. Conclusions drawn might be unreliable.

2 Background

Techniques such as Thiessen polygons, local and global polynomial interpolation and spline interpolation have been used for mapping rainfall fields (Tabios, 1985), but they are not always found reliable in mountainous regions (Creutin and Obled 1982, Lebel 1987). According to Creutin and Obled "none of the statistical methods presented are able to fully account for, both climatologically and spatially, the statistical properties of rainfall fields". But, "in regions with intense and strongly varying rainfall events, sophisticated techniques such as kriging provide a much better estimation than any of the more commonly used techniques". However, if the region is small, with a simple topography and a very dense gauge network, there is no improvement from kriging compared to much simpler inverse methods to map rainfall (Dirks, 1998).

The kriging method was originally developed by Matheron (1971). The method was first applied to mining engineering in South Africa (Journel, 1978). And then to subsurface hydrology, e.g. to the estimations of parameters and network design of wells (Bastin, 1984). Rainfall processes in a river basin can be regarded as a two dimensional random field, since they have a distinct semi-variogram in any arbitrary time interval. Bastin introduced a notion of climatological mean semi-variogram to described the influence of rainfall intensity variation on seasonal change, and then designed rain-gauge networks using a climatological mean semi-variogram.

Althouhg most of the researches assumed stationary random rainfall fields, the universal and disjunctive kriging methods use non-stationary random fields and non-linear estimator respectively. To estimate rainfall in mountainous areas, Chua and Bras (1982) used two spatial mean variation statistical model to calibrate the semi-variogram parameters, i.e. the generalized covariance of order k, and the detrend method. Goovaerts (2000) presented three multivariate geostatistical algorithms for incorporating elevation into the spatial interpolation of rainfall, and compared the prediction performances of the thre geostatistical interpolation algorithms through cross-validation. The spatial relationship can be used to describe the variation of the rainfall process in space, which is described by a single regionalized variable (Isaaks and Srivastava, 1989).

3 Interpolation methods

Three interpolation methods were tested – inverse distance weighting, ordinary kriging and simple kriging.

Inverse distance weighted (IDW) interpolation determines cell values using a linearly weighted combination of a set of sample points. The weight is a function of inverse distance. The interpolation IDW allows the user to control the significance of known points upon the interpolated values based upon their distance from the output points. The best results from IDW interpolation are obtained when sampling is sufficiently dense with regard to the local variation the user is attempting to simulate. If the sampling of input points is sparse or very uneven the result may not sufficiently represent the desired surface. The IDW interpolation is known for its tendency to isolate high values thus providing unrealistic interpolation results.

Kriging method – the estimator Z_K^* is a linear combination of *n* available point-rainfall recordings $Z(x_i)$ located at x_i and with weigtings $\lambda_i Z_K^*$ can be expressed as

$$Z_K^* = \sum_{i=1}^{n} \lambda_i Z(x_i) \tag{1}$$

The optimal weigtings are computed from the block kriging system. The system is obtained by applying the Lagrange multipliers method as follows:

$$\sum_{j=1}^{n} \lambda_j \gamma(x_i, x_j) + \mu = \bar{\gamma}(V, x_i) \quad i=1,2,\ldots,n$$

$$\sum_{i=1}^{n} \lambda_i = 1 \qquad (2)$$

$$\sigma_K^2 = \sum_{i=1}^{n} \lambda_j \bar{\gamma}(V, x_i) + \mu \qquad (3)$$

where $\gamma(x_i, x_j)$ (mm^2) is the semi-variogram of raingauge x_i, $\bar{\gamma}(V, x_i)$ (mm^2) represents the average semi-variogram of estimated area V and rain-gauge x_i, λ_i is the weighting of every rain-gauge, σ_K^2 (mm^2) is the kriging estimated variance and μ (mm^2) denotes the Lagrange multipliers.
The basic experimental semi-variogram is

$$\gamma(t, h_{ij}) = s^2(t)\gamma_d^*(h_{ij}, a) \qquad (4)$$

in which

$$\gamma_d^*(h_{ij}, a) = \frac{1}{2T}\sum_{t=1}^{T}\left[\frac{p(t, x_i) - p(t, x_j)}{s(t)}\right]^2 \qquad (5)$$

where $\gamma_d^*(h_{ij}, a)$ (mm^2) denotes the mean semi-variogram, and is time invariant, h_{ij} (m) represents the distance between arbitrary rain-gauges x_i and x_j, a (m) is the range of the mean variogram, $p(t,x_i)$ (mm) represents rainfall of rain-gauge x_i for time period *t*, *T* (h) is the total duration of all rainfall events, and *s(t)* (mm) denotes the standard deviation of rainfall of all rain-gauges for time period t.

3.1 Comparison of the kriging procedures

The performance of the kriging system may be checked by a cross-validation procedure (Cooper and Istok, 1988). Sample value Z_K^* are deleted from the dataset one at a time and then kriging is performed with the remaining sample values to estimate the value of Z(x_i) at the location of the deleted sample. Statistics of the kriging error (i.e. differences between the sample values and the corresponding leave-one-out estimates for all sample points) are generally used to determine if there is a bias in the estimates and to determine the typice size of error.
The mean errot (ME) checks if the estimation is biased or not (Chua and Bras, 1982):

$$ME = \frac{1}{n}\sum_{i=1}^{n} Z_K^* - Z(x_i) \qquad (6)$$

The root mean square error (RMSE) is a measure of accuracy of the method, where the estimates are considered accurate if RMSE is close to zero:

$$RMSE = \frac{1}{n}\sum_{i=1}^{n}\sqrt{Z_K^* - Z(x_i)^2} \qquad (7)$$

4 Study area – Jeseniky Mountains

The Jeseniky PLA is spread out in the very northern part of Moravia and the Czech part of Silesia and on the frontier between the Moravia-Silesia and Olomouc regions and in the Bruntál, Jeseník and Šumperk districts.
The are includes Hrubý Jeseník and adjacent parts of Hanušovická and Zlatohorská uplands. The relief corresponds to broken mountainous landscape with deep valleys and gradually rounded peaks. From a geological point of view the area is constituted mostly of acid rocks with few nutrients. The dominant soil type is cambisol podsol with mainly humus-ferruginous podsol in high-altitude localities, which is waterlogged and boggy in some parts.

The climate is quite cold. The high-altitude locality is one the coldest in the Czech Republic (on Praděd peak ther is annual precipitation 1440 mm and an average temperature 0,9°C). Anemographic systems are significant feature and had a significant effect on the emergence of glacial cirque and its flora diversity.

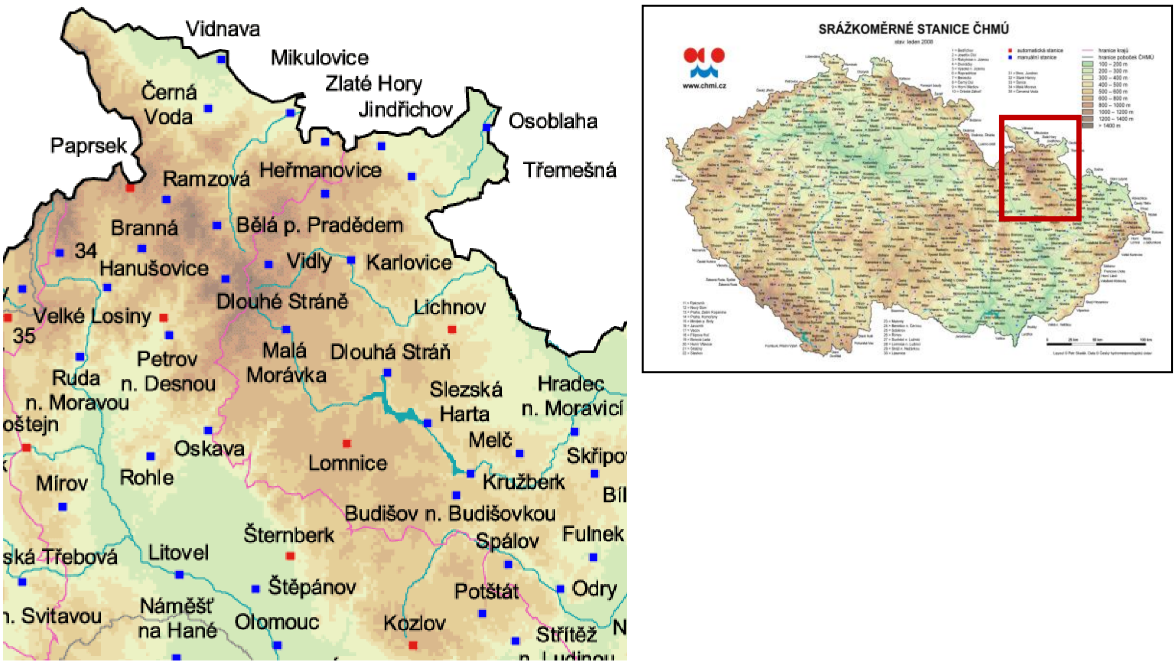

Fig. 1. Rain-gauges in the Jeseniky Mountains (from Czech Hydrometeorological Institute)

5 Data and precipitation uncertainty

The initial input to the analysis is the set of monthly precipitation records for one month (October 2003 – time period 1.10.-12.10.) in Jeseniky Mountains and surrouding areas. The number of climatological stations recording rainfall data has varied, it was used 18 rainfall stations.

The amount of precipitation in a location is commonly measured using a rain gauge. The obtained precipitation is affected by errors and uncertainties, as every instrumental measurement. It is known that there are some well-known error sources that affect rainfall measurement (Michelson, 2004). The possible uncertainties can be classified as measurement errors, systematic uncertainties and random uncertainties.

Measurement errors are usually a consequence of a wrongly located station, with surrounding obstacles preventing an accurate measurement. In such cases, the is not representative of its surroundings. The other possible measurement errors are mistakes in the construction or design of the rain gauge, observer errors or unforeseen erroes. Systematic uncertainties, whose usual outcome is the underestimation of precipitation, are the consequence of the combination of different effects. Among them, we can distinguish the wind flow, affecting the collection of water through the gauge orifice, the evaporation of collected water an the splashing or driftinf of drops when they impact on the gauge.

The uncertainty due to the effect of the wind is the most relevant among systematic uncertainties (Johanson, 2003) and they depend on the wind velocity and direction, the precipitation type and its spatial and temporal variability. Random uncertainties are inherent in the nature of precipitation, a collection of drops, snowflakes or hail with a spatial and temporal size distribution or concentration that is random.

The combined action of the previously decribed sources of error can lead to a recorded amount of precipitation that sensitevely underestimates the real precipitation value (Groisman, 1994). Therefore, the rainfall measured with a rain gauge has associated uncertainties that can be both significant and difficult to evaluate, because they are highly unfluenced by micro-scale factors. Linacre estimates as a typical value an uncertainty of 2 mm fo rainfalls belows 40 mm or 5% relative error for larger precipitation. (Linacre, 1992)

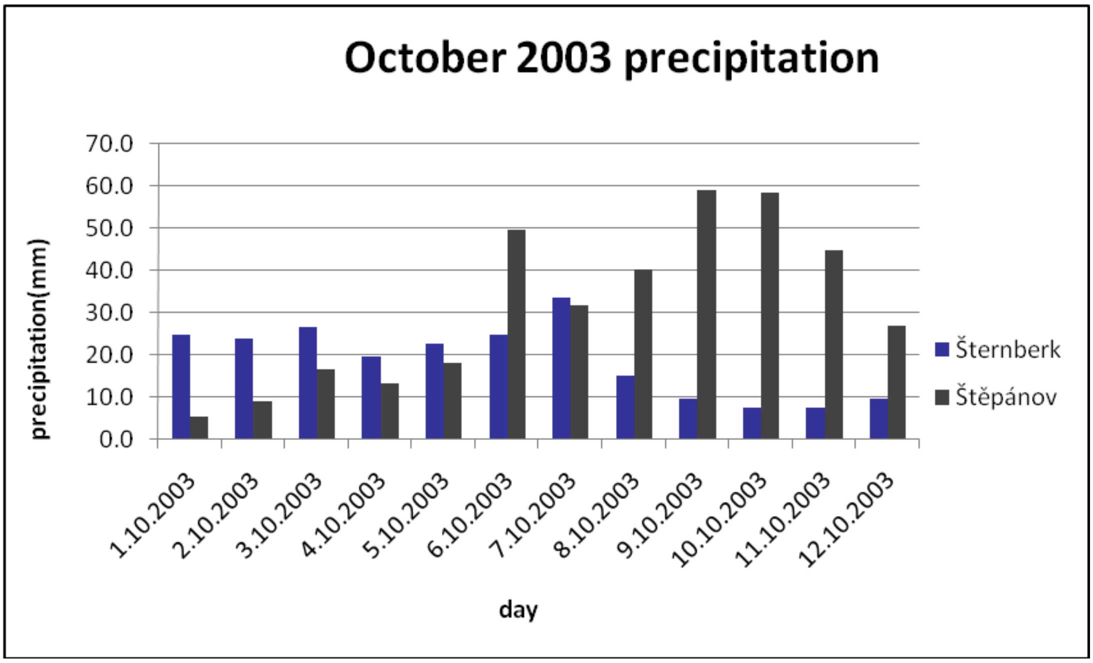

Fig. 2. Examples of rainfall in two rain gauge stations in Jeseniky Mountains

Table 1. Statistical parameters of October 2003 (time period 1.10.-12.10.03) precipitation for rain gauge stations

Rain gauge station	Minimum	Maximum	Mean
Šterberk	7.7	33.5	18.8
Štěpánov	5.4	59.0	31.2
Lomnice	4.0	28.2	15.1
Oskava	6.7	40.5	31.3
Malá Morávka	6.9	43.2	28.6
Petrov n.Desnou	3.2	48.9	29.2
Ruda n.Moravou	4.1	37.5	16.9
Velké Losiny	1.5	29.9	10.8
Králiky	3.1	33.5	11.5
Hanušovice	5.5	41.5	25.6
Branná	6.1	50.2	32.4
Ramzová	4.9	46.9	27.6
Černá voda	5.2	44.5	24.8
Heřmanovice	5.1	40.3	19.5
Mikulovice	3.6	31.2	10.4
Bělá p.Pradědem	5.9	51.4	27.9
Dlouhé Stráně	4.8	48.8	21.0
Karlovice	2.1	36.7	17.8

We can see in Table 1 statistical parameters for time period 1.10.-12.10.03 for all rain gauge stations. In the case of mean maximum precipitation, extreme precipitation are in Štěpánov rain gauge station (59mm), and in the case of mean minimum precipitation it is Velké Losiny rain gauge station.

6 Results

A standard cross-validation procedure is applied to all methods as explained in the chapter 3.1. The root mean square error (RMSE) is a measure of accuracy of the estimate, higher weight is given to larger errors compared to smaller errors, and over and underestimations do not compensate each other as they do with ME. **RMSE** is smaller for ordinary kriging (**3.15 mm**) than for simple kriging (**3.47 mm**) or inverse distance weighting (**3.52 mm**).
This mean that, on average, there are fewer large errors when using **ordinary kriging** than simple kriging or inverse distance weighting.
All three methods were tested in several time steps (15-minutes, 1 day, whole time period).

Table 2. Cross validation for measurement 15-minutes (1.10.2003 0:15)

Rain gauge station	Record [mm]	IDW [mm]	Ordinary kriging [mm]	Simple kriging [mm]
Šterberk	1.5	1.8	1.6	1.4
Štěpánov	3.8	3.5	3.5	3.9
Lomnice	2.1	1.8	2.3	2.4
Oskava	0.8	1.2	0.9	1.2
Malá Morávka	0.9	1.2	1.0	1.3
Petrov n.Desnou	1.9	2.1	1.8	1.7
Ruda n.Moravou	2.8	2.9	2.7	2.5
Velké Losiny	1.2	1.0	1.4	1.1
Králiky	4.6	4.2	4.7	4.1
Hanušovice	3.9	3.6	3.7	3.8
Branná	3.8	3.9	3.9	3.9
Ramzová	5.7	4.6	5.1	4.9
Černá voda	4.5	4.1	4.1	4.2
Heřmanovice	2.9	3.3	3.2	2.5
Mikulovice	3.1	3.2	3.3	3.4
Bělá p.Pradědem	4.7	4.1	4.6	4.1
Dlouhé Stráně	4.9	4.2	4.5	4.4
Karlovice	4.2	4.5	3.9	3.8

7 Conclusion

The spatial and temporal variation of day precipitation was investigated for the Jeseniky Mountains. The many types of records of precipitation (15-minutes, day, whole time period) from various meteorological stations were used.
The multivariate geostatistical approach was used with the cross-validation method to estimate the hourly data, day data, which was assumed to be unknown. From the results of estimation and validation, the above approach seems to produse satisfactory results, but i will be very usefull to try another interpolation method like cokriging.
This approaches presented here can be helpful in hydrological analysis of a flood simulating and forecasting system, especially for real-time rainfall estimations.

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