Hypotheses testing in regression models and their using in landslide detection

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Abstrakt. Testování hypotéz v regresních modelech a jejich využití pro detekci svahových pohybů. Stanovení velikosti svahových pohybů na základě údajů měřených v delším časovém období pomocí jedné metody a jednoho typu přístroje se zdá být jednouchým úkolem, ve kterém jsou na základě známých vlastností měřicího přístroje, a tím i známé přesnosti měření, jasně označeny body příp. oblasti, které se pohybují vůči vhodně vybraným stabilním bodům. Z hlediska statistiky už tento problém takto jednoznačné řešení nemá, protože některá, na první pohled odchýlená měření, nemusí být považována za posun a naopak i malé rozdíly v poloze bodu mohou v některých případech znamenat pohyb. U těchto sporných případů může statistika pomoci zkvalitnit analýzu svahových pohybů pomocí teorie regresních modelů a testováním hypotéz v těchto modelech. Hypotézami jsou zde myšleny výroky o stabilitě nebo pohybu jednotlivých bodů, které mohou mít vliv na průběh regresního modelu. Tímto způsobem je pak možné prokázat signifikantní rozdíly v dlouhodobé poloze bodu a s větší pravděpodobností rozhodnout, je-li měřená oblast nebo bod stabilní či se pohybuje. Příspěvek shrnuje poznatky o vývoji modelu a poskytuje základní informace potřebné k jeho pochopení. Hlavními přednostmi modelu jsou jeho relativní nezávislost na typu svahového pohybu, netradiční postup při vyhodnocování měření, schopnost identifikovat jednotlivé složky sesuvu díky deformační kružnici a následně i možnost zjištění tvaru smykové plochy. Postup byl vyvíjen a testován na měřeních pocházejících z dvouletého sledování plošného sesuvu v oblasti karpatského flyše, ale po úpravě parametrů je použitelný i pro jiné typy svahových pohybů a stává se tak vhodným nástrojem k jejich sledování a analýzám případně jako nástroj predikce dalšího vývoje.

Klíčová slova: statistika, testování hypotéz, ANOVA, model měření vektorového parametru, měření sesuvů, konfidenční elipsy

Abstract. Slope displacement monitoring seems like a simple task, concerning the present-day technology and methodology utilized to handle it. Basically, the procedure demands accurate and frequent measurements of relative displacements between active ground points versus stable ones, which is seemingly unambiguous and explicit problem. Nevertheless, some measurements, although they could look like an obvious deviation, do not have to represent shifting of the measured point, while even small change of the position could mean a significant shift in some cases. Statistical methods can help to improve landslide analyses, especially in questionable cases. Improvement can be reached by using the theory of regressive models and hypothesis testing in these models. Herein, the hypotheses state how stability or instability of individual points may affect the course of the regression model which describes the shift. In this way it is possible to prove significant differences in the long-term position of the point and therefore, easier to assess whether the point or the area turns stable or unstable. This paper summarizes findings about model development and also provides basis to better understand the model. Main advantages of this approach are relative independency on the type of slope movements, unconventional procedure for measuring evaluation, ability to identify individual components of the landslide and determine slip surface by virtue of the deformational circle construction. This method was developed and tested on data after two-year surveillance of the landslide in Carpathian flysch belt. It is also presumable that after the modification of parameters the same model can be used for similar type of phenomena and become a suitable instrument for observing and analyzing or predicting landslides.

Keywords: statistics, testing of hypotheses, ANOVA, model of measuring of vector parameter, landslide’s measuring, confidence ellipses,

1 Introduction

Analysis of landslides and slope movements requires careful processing of measurements. Different theoretical approaches and measurement techniques provide a variety of outputs. Outputs are necessary to be processed in order to statistically establish whether the assessing of the slope truly represents the landslide behaviour or just measurement error. Previously mentioned accuracy is a pivot question. Determination of the size of slope movements based on a long-term measurement that uses common technology and methodology appears to be a simple task. All of instruments and methods characteristics are known, as well as the measuring precision. This procedure divides measured points into two groups, the first, containing only moving points and the second containing stable ones. The stability of each point is determined in relation to the fixed ground points (surveying marks), i.e. by comparing distances between measured and fixed points in time. This problem doesn't have ambiguous solution from statistical point of view. Some of measurements, although they could look like obvious deviation, do not have to be considered as shifts of the measured point. On the contrary, even small change of the position could mean significant shift in some cases. Statistical methods can help to improve analyses of landslides, particularly in questionable cases. Improvement can be reached by using the theory of regressive models and hypothesis testing in these models. Hypotheses are claiming whether the point behaviour affects the course of the regression model of point displacements. In this way it is possible to prove significant differences in the long-term position of the point, allowing more accurate prediction of point's stability.
A main objective of the research and this paper is a statistical evaluation of the point mobility (relating to overall landslide stability) with an emphasis on the characteristics of variability, confidence ellipses and hypotheses testing. Statistical hypothesis testing helps to refine parameters of the regression model which describes a chosen slope movement. The deformational circle is considered as the visualization of results of the regression model and describes size and direction of control points' movement. Further, the deformational circle allows finding out the evolution of the landslide in time. Advantage of this approach is possibility of observing main processes in the landslide. These processes are deformation, translation and rotation and all of them affect the shape and position of the deformational circle. Slip surface can be indicated by deformational circle which expresses the size and direction of the movement of indicating points. If these points were deployed properly, then the shape of a deformational circle is more appropriate orthogonal projection of the slip surface.

2 Problems of slope movements and their measurements

Landslides are a serious natural hazard threatening human lives and dwellings. Houses can be damaged as well as communication, infrastructure and other objects standing in the way of a landslide in progress. Structure of the landscape and also terrain is always disrupted. In our country, slip type of landslides [12] are dominant slope processes, followed by some minor occurrences of debris or mud flows, rock fall, creeping or toppling (Fig. 1).
The slope movements are caused due to unbalancing of rock masses when the effect of driving forces (gravity, hydrodynamic pressure ...) prevails over resisting forces which try to prevent movement (strength of rock, friction). The result of the prevailing driving forces is then the slope deformation.
According Záruba [12] this is one of the basic assumptions of slope movements, except man-made interference into the natural development of the slope, natural conditions, which are given by geological, climatic, hydrogeological and geomorphological conditions. These conditions may support materials movement on the slope or inhibit it, and therefore can be classified either as favourable or unfavourable. Among the factors affecting the slope movements and deformations the most important are: changes in the slope angle and height, changes in the water content, the effect of groundwater, frost action and weathering of rocks, changes in vegetation cover, embankment overload, piles, shocks and vibrations. These factors may be further classified, in accordance with their origin (natural or anthropogenic).

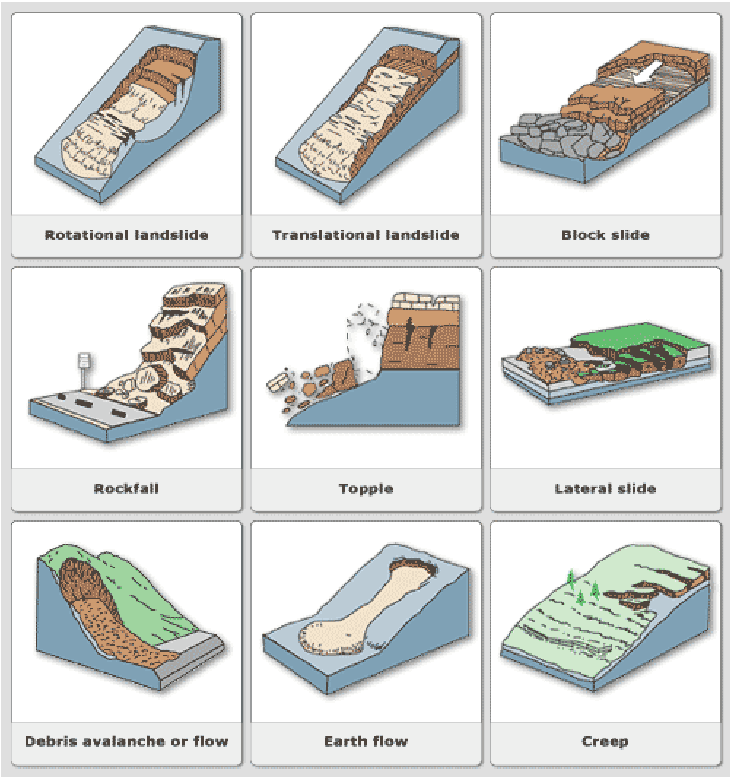

Fig. 1 Possible classification of the slope movements [13]

2.1 Measuring of a slope movement

There are a numerous approaches to the measurement and analysis of slope movements. Some approaches use the monitoring on the basis of relative displacements, i.e. by triangulating between measuring and "fixed" ground points. Other methods use e.g. buried sensors, which can evaluate a shift by virtue of its position and orientation. The possibility of the slope instability may be detected indirectly using instruments that measure phenomena which contribute to this instability. Another way to detect landslide susceptibility of the slope could also be presented by models that calculate a stability factor for the selected slope. These models which are also possible to implement into GIS (Fig. 2), calculate both, the terrain features, as well as external influences (e.g. rainfall).

This paper will deal with methods of geodetic surveying and processing of the data, which are according to Záruba [12], compliant when measured movements are greater than error of the measurement device. For this purpose, network of measured points is set out on the surface of the landslide. This network has to be properly attached to fixed points outside the landslide. Positions of the points have been repeatedly measured either by regular time sequences or after specific events (e.g., spring thaw, intense precipitation, etc.). These methods are also improved by laser rangefinders, which are included in modern total stations recently.

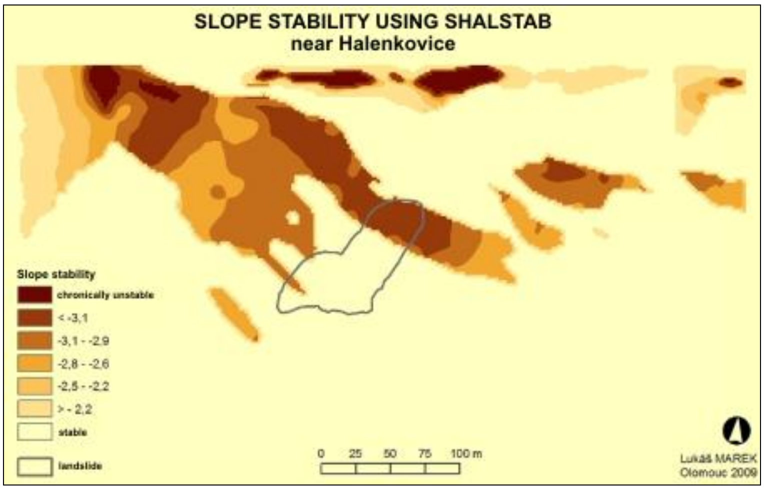

Fig. 2 Stability of slopes near Halenkovice computed in SHALSTAB [9]

3 Model area

The area of interest is situated near the village Halenkovice, in the western part of the homonymic cadastral territory. It lies in the Zlínský kraj, in the former district of Zlin, about 4 km from Napajedla (Fig. 3).

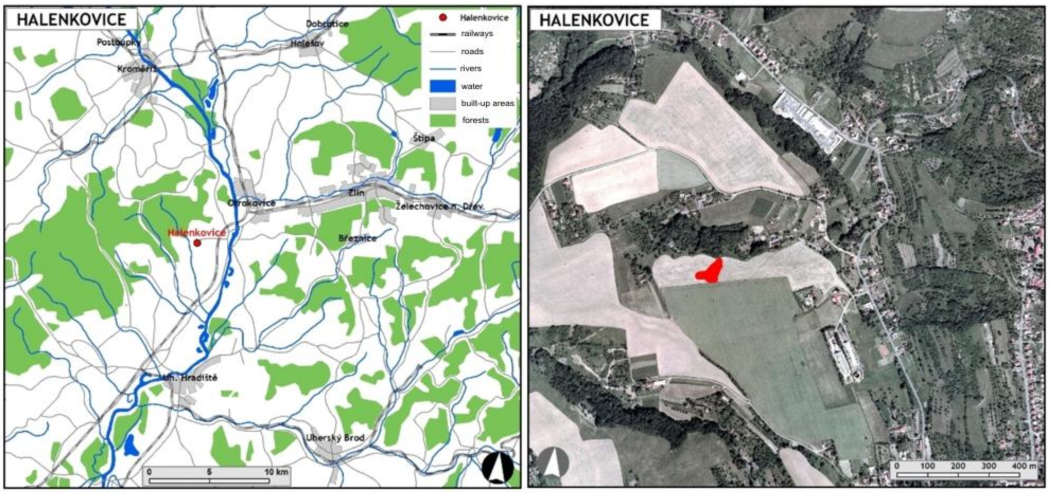

Fig. 3 Location of Halenkovice

3.1 Geology and geomorphology

The territory is located in the outer part of the Western Carpathians which are made from the Mesozoic and Tertiary deposits, so-called flysch Carpathians. It is part of the račanská tectonic stratigraphic unit of the magura flysch group from regional geological point of view. Geological structure of the flysch is characterized by alternating sandstones and conglomerates with clay slates, i.e. layers with variable thickness and different strength characteristics, which goes in favor of instability. [10] Different types of instability and types of slope movements are recorded in the flysch, e.g. shallow landslides that evolve in the soil mantle and their origin is sufficient to slide by increased precipitation amount.
The modeled area is represented by slope with inclination 10-15°, which has NW orientation and has been devastated by recent deformations. Investigated territory is located in the range of altitudes around 250-280 m (Fig. 4).

Shallow landslide was chosen as the model which chiefly corresponds to the mechanical behaviour of the slope. Substantial activity (first failure) dates back from March 2006. Since it has not been properly stabilized further evolution i.e. reactivation stages are expected. A landslide lies in the pasture and it is not near any residential or other buildings, hence there are no people under immediate danger, implying that stabilization is not necessary. Area of the slope deformation is approximately 4400 m^2; its dimensions reach a size 80 m and 70 m. All three zones of landslide progression (detachment, transport and storage) are easily recognized within this landslide. Scarp of the landslide varies in size and height, ranging from 30 to 200 cm. Debris thickness at the toe reaches as much as 50 cm. There are 1 m high lateral accumulations on flanks of the landslide (Fig. 4).

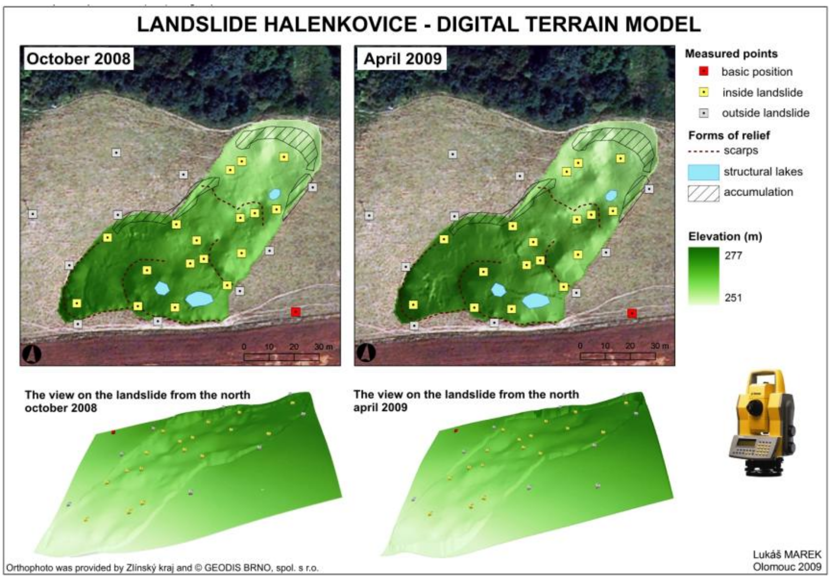

Fig. 4 Model area [9]

4 Statistical methods for determining the slope shift

4.1 Confidence ellipses

Inaccuracy of the measurement can be distinguished from the changes in the measured position of the point using confidence intervals. Confidence interval is the area within which measured points could be representing the measurement error, with certain probability (confidence). Thus, deviation in a series of measurements immediately involves changing of the position of measured points. The method proposed here is based on the analysis of measured data and on the detection of point's variability in time. This variability is statistically calculated as the standard deviation of measured series or group of measurements. The output is then the interval of values (graphically represented by area) which can help to decide whether the data may be characterized as shift displacement or data are only the image of stable point in the confidence tolerance.

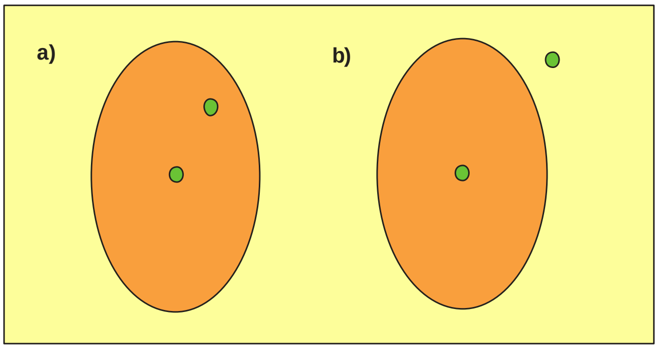

Fig. 5 a) stable point b) active point [9]

An important criterion for assessing the stability of the point using the confidence interval is a time sequence of measurements. If there is one measurement among measurements with the same or very similar average position, which shows clear shift of the point, then it is wrong measurement because of supposed irreversibility of the process, i.e. once changed value cannot return back to its original position (Fig. 5). The paper deals with two-dimensional space, namely confidence ellipses (confidence circles in an ideal case). Parameters of the axes size are given by standard deviation of coordinates X and Y in combination with the device error.
After the plotting of confidence zones for the reference point in different time sequences, the parameters of the confidence ellipse could be assessed. If the ellipse parameters are not identical and did not even overlap each other, then there has been a real shift displacement of the measured point because the error could not been covered by the maximum confidence zone. Assuming normal distribution of the phenomenon, confidence interval based on the standard deviation can capture up to 68% of the phenomenon, twice the standard deviation up to 95% of the prhenomenon and three times standard deviation even more than 99% (Fig. 6). Therefore, selection of the threshold criteria is extremely important for the assessment.

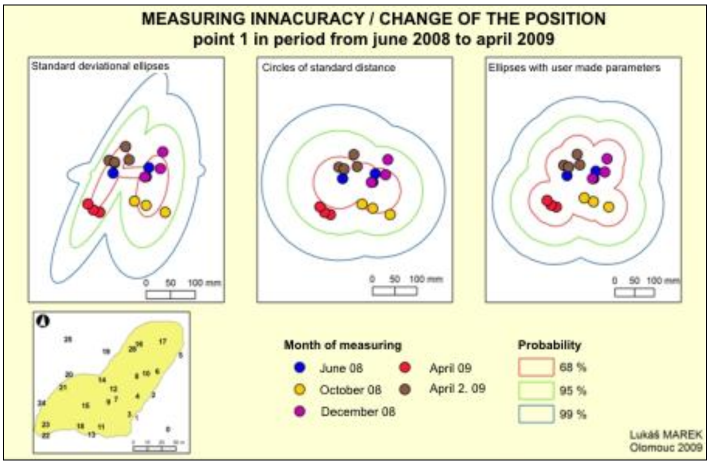

Fig. 6 confidence ellipses [9]

4.2 Statistical evaluation - the characteristics of variability

Characteristics of variability were used primarily for detecting characteristics of the measured data. As the most important feature the standard deviation was selected. This enables the measurement accuracy evaluation, and more importantly (in this research at least) pointing to the cases of probable shift displacements.
The most important characteristics of variation in statistical datasets according to Brázdil [1] are variance and standard deviation. Error of measuring instruments is often placed as the standard deviation, so it is very frequently mentioned concept in this paper.
Variance $\mathbf{s^2}$ from n values x_i is the average of squares of deviations of individual values from their arithmetic character average, defined as:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n} \quad \text{or} \quad s^2 = \overline{x^2} - \bar{x}^2$$

The square root of the variance – the standard deviation **s** is more used in practice. The standard deviation is a measure of variability of values x_i of random variable around the average.

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$

Tab. 1 The characteristics of variability (coordinates)
(values in red could indicate active points because of severalfold bigger values compared with other points)

Point	Standard deviation (cm)				Variance (cm^2)			
	Y	X	Z	Sum	Y	X	Z	Sum
4	18,87	21,84	12,76	27,71	356,26	477,15	162,80	767,95
5	5,53	4,25	1,20	5,30	30,62	18,05	1,45	28,14
6	37,15	57,54	14,00	67,78	1380,38	3311,35	195,86	4594,16
7	3,70	6,95	0,95	7,26	13,70	48,34	0,91	52,69
13	4,14	3,86	1,25	4,21	17,13	14,88	1,57	20,26
14	21,83	15,90	5,62	23,28	476,35	252,91	31,60	541,84
16	55,60	108,85	24,98	122,12	3090,84	11849,20	624,22	14913,78
17	48,02	72,55	12,34	85,87	2306,34	5263,60	152,20	7373,03
28	4,45	9,99	2,12	10,27	19,80	99,80	4,50	105,48
0	3,25	4,32	3,40	4,86	10,59	18,63	11,56	23,58

Tab. 2 The characteristics of variability (angle and distance)
(values in red could indicate active points because of severalfold bigger values compared with other points)

Point	Standard deviation		Variance	
	Distance (cm)	Horizontal angle (°)	Distance (cm^2)	Horizontal angle (°)
4	5,90	0,585	43,48	0,383
5	3,47	0,033	13,65	0,001
6	54,13	0,662	3242,04	0,485
7	2,22	0,087	5,33	0,005
13	2,97	0,031	7,56	0,001
14	7,89	0,288	76,13	0,086
16	88,54	0,814	8349,21	0,723
17	73,34	0,496	6126,95	0,279
28	6,76	0,069	36,24	0,003

5 Model of measurement of vector parameter

As stated by Marek [8], it is possible to use the regression models, designed by Kubáček [4] or differential operators of vector calculus as tools for monitoring changes in spatial location of points in the steep terrain (with suspicion of the possibility of sliding). Marek in [8] also proposes the use of multiepoch regression models that considers the possibility of changes in the position of observed points. The construction of the regression models for the landslide detection are made with the use of [3], [5] a [6], [7].

Linear changes in positions of points in time are estimated by the calculation of the unknown values of the parameters in the given models. It means that velocity of the “shift” in observed points is estimated. This will result also in an estimation of the variability in position changes. The null hypothesis (the shift parameter is equal to zero) can be tested at each point with the given theory of linear regression models [4]. The alternative hypothesis may also consider the use of rotation and deformation tensor. The purpose of the hypothesis testing is to refute the null hypothesis (H_0) and confirm alternative hypothesis (H_1 or H_A). Only in case, when the hypothesis proves to be untenable, it is possible to talk about unambiguous test result [2].

Before alternatives will be investigated in more details, look at the table that shows points at which the individual shifts are statistically significant (results of the regression models).

Tab. 3 Results of hypotheses testing
(cross – active points ANOVA , double-cross – active point proved by improved method – regression model)

Point	Test p < 0,05			Point	Test p < 0,05		
	Y	X	Z		Y	X	Z
1				15			
2			x	16	xx	xx	xx
3				17	x	x	x
4	xx	xx	xx	18		x	x
5	x		x	19			x
6	xx	xx	xx	20			x
7				21	x		
8				22		x	
9	xx	xx	xx	23	x	x	
10		x	x	24		x	x
11		x	x	25			x
12				26	x	x	
13			x	27		x	
14	x	x	x	28	x		x

5.1 Model 1 – Regression model

Goal of the paper is to find the estimators of the velocity shifts in the *x* and *y* plane coordinates. The coordinates are measured in epochs repeatedly (with the exact period). Measurements are located in places where the geological and geophysical phenomena which lead to the change of position, are expected. These assumptions allow us to construct the regression model. The model could not be numerically reliable and it would be difficult to construct reliable model at all without these assumptions. We will therefore assume that the change of coordinates is a linear function of the measurement and reference measurement.

Nine points were used for the construction of the regression model due to problems with continuous measurement (results of measurement were available at each period of experimental measurement). The experiment will then be modeled by the linear regression model. Denote θ_0 as measurements of coordinates in the reference epoch. For the calculation we will consider the accuracy of the coordinates measurement $\sigma^2 = 0.001^2$ m. Furthermore, we will assume that the coordinates in other epochs are given by the relationship

$$\mathbf{Y}_t \quad = \quad \mathbf{\Theta}_0 + \mathbf{v}(t - t_0) + \boldsymbol{\varepsilon}\,,$$

where *v* is the velocity of the shift. The aim of the regression model is an estimation of the parameter value in each measured point. The next goal is to test (with the use of the theory of hypothesis testing) the null hypothesis

$$H_0: \quad \mathbf{v}_j = \mathbf{0}\,.$$

The exact analysis of this model is not the aim of our contribution here. The study is made in Marek [8]. For our study we will use the idea based on translation, rotation and deformation coordinates using regression model with transformation matrix $\boldsymbol{T}$.

5.2 Model 2 – Regression model with transformation matrix *T*

Next the alternative model is assumed, where is dealt with translation, deformation and rotation of coordinates. Let the regression model is considered

$$\mathbf{Y} \quad = \quad \mathbf{t} + \mathbf{T}\mathbf{X},$$

where ***X*** is the matrix of coordinates at the beginning of the measurement and ***Y*** is the vector of coordinates of measured points in the next stage. The transformation matrix T can be decomposed into a deformation matrix $\boldsymbol{D_1}$ and rotation $\boldsymbol{Q_1}$, or $\boldsymbol{Q_2}$ rotation and deformation $\boldsymbol{D_2}$ in the form:

$$\mathbf{T} \quad = \quad \mathbf{D}_1\mathbf{Q}_1 = \mathbf{Q}_2\mathbf{D}_2.$$

The matrix ***TT'*** can be also (with the help of singular value decomposition – known as SVD algorithm [11]) decomposed as

$$\mathbf{T} \quad = \quad \mathbf{U\Lambda V} = (\mathbf{u}_1, \ldots, \mathbf{u}_k) \begin{pmatrix} \lambda_1, & 0, & \ldots & 0, & 0 \\ & & \ldots & & \\ 0, & 0, & \ldots & 0, & \lambda_k \end{pmatrix} \begin{pmatrix} \mathbf{v}'_1 \\ \ldots \\ \mathbf{v}'_k \end{pmatrix}.$$

The matrix ***Λ*** is the deformation matrix. The detailed analysis and numerical study of the model mentioned above is made and proved in Marek [8]. Only the numerical and graphical visualization of obtained results were used for our contribution (Fig. 7).

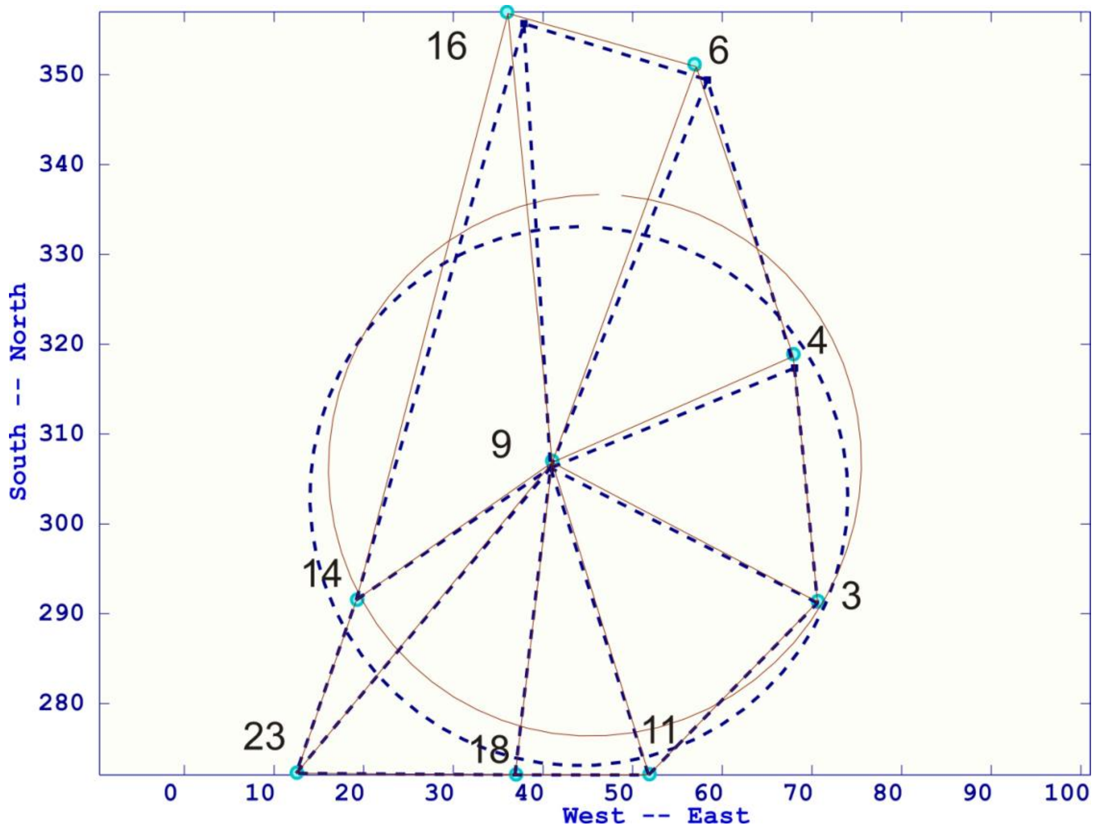

Fig. 7 Results for the regression model with the transformation matrix.

It is necessary to mention that estimated shifts were also simulated by the circle, which was designed and constructed in the first stage of the measurement in order to describe the overall situation. The translation, rotation and deformation of the circle in the last stage (computed with the use of estimated shifts), can be seen also in the picture mentioned above.

6 Conclusions

We need to realize that statistical evaluation is not "omnipotent" and we should verify information either in situ or by other methods. Even points which were primarily evaluated as active points were finally evaluated only as an image of permanent points. Evaluation of Z coordinates (altitude) may be difficult in some cases. Because even if the position of points is changed, then altitude didn't have to change or changes could be in the order of only a few millimetres which can cause a small dispersion. Result of this can be that even very small inaccuracies are assessed as identifiers of the shift of points

in the vertical direction. Therefore it would be ideal to examine whether a particular source date had significant influence, i.e. is it justifiable to treat it as displaced or stable by the proposed model.
Characteristics of variability (standard deviation and variance) and confidential ellipses resulting from them are used for distinguishing stable point from active points in this paper. Methods of statistical hypotheses testing are also used for the evaluation of measuring points' stability. Individual coordinates of control points which were gained in different epochs, were used in these methods. In order to achieve the best possible results, the model of the vector parameter is involved into the testing. This model describes measurements of control points in different epochs and so it is used in testing hypotheses about a compliance of averages.
Described model is able to distinguish motion points from the inaccuracy of measuring. It is caused by using of multiepoch measuring and statistical hypotheses testing. An important finding is the possibility of using the results of regression model and their visualization to identify the individual parts of the landsliding process. The deformational circle serves this purpose. It is an indicator of changes in the position of points and thanks to it is possible to identify a slip surface of the landslide. Modeled slip surface corresponds to the reality better when the set of measured control points was placed appropriately. The use of decomposition using SVD (Singular Value Decomposition) is an unconventional element in the development of the regression model.

Although the model was developed and also tested on the specific area of Carpathian flysch, it is generally also applicable for other types of slope movements. But another calibrations and testing, which would adapt model for local conditions, are necessary. These adjustments should be based on previous measurements or a good knowledge about processes on the area of interest.

This contribution was supported by project of Czech Science Foundation GA CR 205/09/1159 --"The Intelligent System for Interactive Support of Thematic Map Design".

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