PRECISE EVALUATION OF PARTIAL DERIVATIVES AS AN ADVANCED TOOL FOR TERRAIN ANALYSIS

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Abstrakt

V mnoha GIS nalezneme nástroje pro základní analýzu georeliéfu jako je výpočet sklonu, orientace a různých křivostí. Pro pokročilé analýzy terénu jsou však zapotřebí vrstvy počítané pomocí parciálních derivací 3. řádu. Výpočet parciálních derivací 3. řádu je však citlivý na vstupní data (přesnost a hladkost digitálního modelu terénu) a numericky náročný a proto jeho implementace ve většině GIS chybí. V rámci tohoto příspěvku představíme metodu numericky stabilního výpočtu parciálních derivací 3. řádu, založenou na aproximaci georeliéfu polynomiální plochou 3. řádu a využívající váženou metodu nejmenších čtverců (MNČ). Klíčovým přínosem tohoto příspěvku je představení osmi různých vah použitých při výpočtu parciálních derivací třetího řádu pomocí MNČ. Testujeme zde 9 metod (neváženou MNČ a 8 vah pro váženou MNČ) na modelovém reliéfu, který je daný analytickou funkcí. Testovací funkce je vytvořena tak, aby co nejlépe napodobovala chování georeliéfu - na daném intervalu obsahuje vrcholy, hřbety a sedlové body. Díky explicitnímu vyjádření testovací funkce jsme schopni určit přesné hodnoty derivací v libovolném bodě, což nám umožňuje využít tuto testovací plochu jako etalon. Přesnost aproximace se pak určuje jako statistické vyhodnocení diferencí etalonového povrchu vybrané parciální derivace a povrchu parciální derivace stejného typu aproximované pomocí představené metody. Využití parciálních derivací 3. řádu nalezneme např. u poloautomatické segmentace georeliéfu s využitím vymezování elementárních forem georeliéfu, které je součástí budovaného Geomorfologického informačního systému. Zde se parciální derivace 3. řádu využívají pro automatické určení segmentů hranic jednotlivých elementárních forem.

Klíčová slova: parciální derivace, třetí řád, test kvality.

Abstract

In many GIS we can find tools for basic georelief analyses as the slope and orientation computation, including basic types of curvatures. For advanced terrain analyses are required layers computed from the 3rd partial derivatives. The computation of the 3rd partial derivatives is sensitive to input data (accuracy and smoothness of digital terrain model) and numerically instable and thus is not implemented in the most GIS. Within this article we introduce numerically stabile method for computation of the 3rd order partial derivatives using the weighted least square method (LSC). The most important contribution of this paper is introducing 8 various weights into the LSC evaluation of derivatives up to third order. We are testing 9 methods (unweighted LSC and 8 weights) on a model relief given as an analytical function. The test function is created so it emulates the behavior of a natural georelief – in defined interval it creates peaks, ridges and saddle points. Thanks to the explicit representation of the test function we are able to define the exact derivation values in any point. This enables usage of the test function as the etalon. The approximation preciseness is then evaluated using statistical methods on differences of the two tested surfaces – the etalon and the surface of desired partial derivative approximated by the introduced method. The usage of the 3rd order partial derivatives may be for example found at semi-automated georelief segmentation using the delimitation of elementary forms of georelief as part of Geomorphologic Information System. Here are the partial derivatives used for automatic delineation of elementary forms boundaries.

Keywords: partial derivation, 3rd order, quality test.

INTRODUCTION

Common GIS are not offering evaluation of partial derivatives of the 3rd order. Precise partial derivatives of the 3rd order are crucial for computing morphometrical variables of the 3rd order widely used in geomorphometry (Minár, (2008), Krcho, (2001)). For the requirements of Geomorphologic Information System (GmIS) was in Pacina (2008) implemented a robust algorithm for approximation of partial derivatives up to the 3rd order with sufficient quality. This algorithm was based on weighted least squares method, however, the effect of weights was not studied in depth. This article is focused on testing the accuracy of this method for various weights on a test polynomial function with characteristics of a topographical surface. Analytical expression of the test function allows us to compute absolutely precise partial derivatives which are further used as the *etalon*.

Approximated partial derivatives by method described in Pacina (2008), Jenčo, et al. (2009) and Pacina (2009a) are used for computation of derived morphometrical variables up to the 3rd order[1]. Surfaces of derived morphometrical variables of different orders are within the GmIS further on used for automatic delimitation of elementary forms of georelief (see Pacina (2008), Pacina (2009), ,Minár (2008), Tachikawa et al. (1993), Pike (1988)). The usage of precise partial derivatives is overall much wider. The most important contribution of this article is the study of the effect of various weights in the LSC adjustment, which was not yet studied. In all published papers are the weights either not introduced at all or it is used only one weight system without any elaboration of setting up its parameters.

Proposed method can be used for computing further morphometrical variables, namely maximal curvatures of topography with further applications e. g. in hydrology (O'Callaghan, J.F., and Mark, D.M. (1984), Chang, Y.C., Song, G.S. and Hsu, S.K. (1998) or Riazanoff, S., Cervelle, B., and Chorowicz, J. (1988)) or other geomorphologic analysis using the morphometrical variables of higher orders (Tremboš et al. (1994)).

METHODOLOGY

Tested methods for derivatives computation

This algorithm approximates the partial derivatives from a digital elevation model (DEM) given in a form of a raster. It interpolates a 5x5 neighborhood by general polynomial surface of the 3rd order, using the weighted linear least squares method. The general polynomial surface of the 3rd order is given by:

$$z_{i,j}(x,y) = a_0 + a_1(x-x_i) + a_2(y-y_j) + a_3(x-x_i)^2 + a_4(y-y_j)^2 + a_5(x-x_i)(y-y_j) \\ +a_6(x-x_i)^3 + a_7(y-y_j)^3 + a_8(x-x_i)^2(y-y_j) + a_9(x-x_i)(y-y_j)^2 \qquad (1)$$

This formula allows to build one equation for each point with coordinates (x, y) lying in the neighborhood of the point (x_i, y_j) where we want to compute the derivatives. Because have used the 5x5 neighborhood of actually computed point, we can build a linear system of 25 equations and 10 unknowns a_0 … a_9. The linear system is expected to be overdetermined so we have solved the system by the least squares method.. On **Chyba! Nenalezen zdroj odkazů.** are shown the nodes of the 5x5 neighborhood. Symbols *f* in each of the nodes represents function values in the node. Value *h* is distance between the nodes.

[1] For example the morphometrical variables of the 3rd order: a_{gn} - change of orientation change in the direction of a fall line, A_{Ntt} – change of orientation in the direction of a contour line.

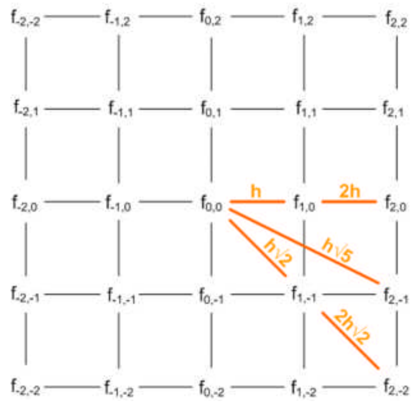

Fig. 1. Nodes of the 5x5 neighborhood for $(x_i, y_j) = (0, 0)$

Estimation of derivatives

Once having coefficients $a_0 \ldots a_9$ of the polynomial surface for particular point (x_i, y_j), we can approximate the derivatives in the point (x_i, y_j) as derivatives of the polynomial surface. For example, partial derivative of *z* by *x* then would be:

$$\frac{\partial z}{\partial x} = a_1 + 2a_3(x - x_i) + a_5(y - y_j) + 3a_6(x - x_i)^2 + \\ +2a_8(x - x_i)(y - y_j) + a_9(y - y_j)^2. \quad (2)$$

From which results:

$$\left.\frac{\partial z}{\partial x}\right|_{(x_i,y_j)} = a_1. \quad (3)$$

And the other derivatives are:

$$\left.\frac{\partial z}{\partial y}\right|_{(x_i,y_j)} = a_2, \left.\frac{\partial z^2}{\partial x^2}\right|_{(x_i,y_j)} = 2a_3, \left.\frac{\partial z^2}{\partial y^2}\right|_{(x_i,y_j)} = 2a_4, \left.\frac{\partial z^2}{\partial x \partial y}\right|_{(x_i,y_j)} = a_5,$$

$$\left.\frac{\partial z^3}{\partial x^3}\right|_{(x_i,y_j)} = 6a_6, \left.\frac{\partial z^3}{\partial y^3}\right|_{(x_i,y_j)} = 6a_7, \left.\frac{\partial z^3}{\partial x^2 \partial y}\right|_{(x_i,y_j)} = 2a_8, \left.\frac{\partial z^3}{\partial x \partial y^2}\right|_{(x_i,y_j)} = 2a_9 \quad (4)$$

Weights

In unweighted least squares, all 25 points from the 5x5 neighborhood area influenced the results (third derivatives) in the same way. Therefore, the result is affected much more by 16 points that are at the boundary of the 5x5 neighborhood area than by 9 points which are close to the computational point. This fact has a smoothing effect on the derivatives computed by unweighted least squares, because distant points have together bigger weight than closer points. To encounter the higher influence of points closer to the center of approximate area, we have used the weighted least square method. We have proposed to weigh systems with weights w^ε and w^δ. Both of them give bigger weights to points closer to the computational point (center of 5x5 area) and both depend on some parameter. The first weight w^ε is given by

$$w_{i,j}^{\varepsilon} = \frac{\varepsilon + 2h\sqrt{2} - h\sqrt{i^2+j^2}}{2h\sqrt{2}} \quad (5)$$

Where ε > 0 is a parameter, $w_{i,j}$ is the weight of point x_i,y_j with respect to point x_0,y_0 and h is the distance between points x_0,y_0 and x_i,y_j. The second weigh is given by

$$w_{i,j}^{\delta} = \frac{2h\sqrt{2}}{\delta + h\sqrt{i^2+j^2}} \quad (6)$$

where δ > 0 (for example 0.1), which influences the importance of the points further from the center.

Both weight systems were developed to take into account the influence of the surrounding nodes, which should be decreasing with the increasing distance from the middle. Weight functions w^{ε} and w^{δ} for various parameters ε are δ and for h=1 are plotted at **Chyba! Nenalezen zdroj odkazů.** and **Chyba! Nenalezen zdroj odkazů.**.

The weights affect the coefficients of the third order polynomial during the adjustment process in the optimization function. The only difference is that instead of using identity weight matrix, a diagonal matrix is used.

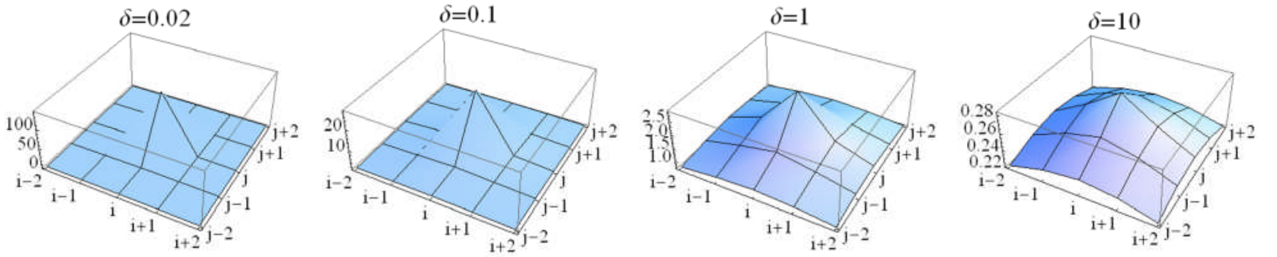

Fig. 2. Weight w^{ε} influence on the surroundings points

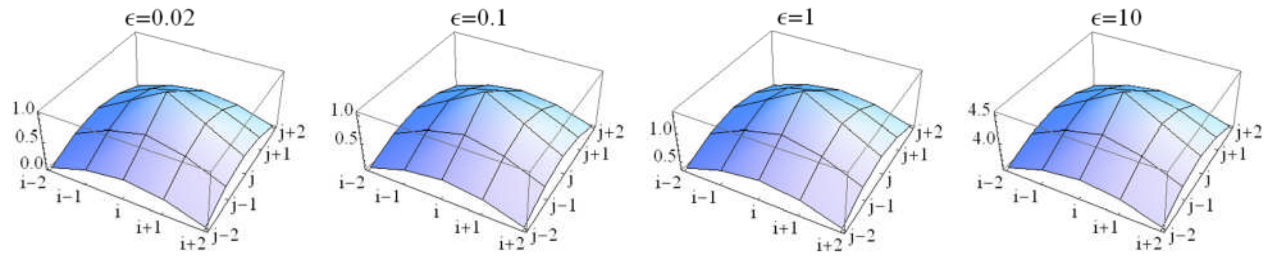

Fig. 3. Weight w^{δ} influence on the surroundings points

Least squares solution

The system of linear equations $\mathbf{Qa} = \mathbf{f}$ for computation of the unknown coefficients $\mathbf{a}$ can be overestimated hence in general must not have any solution and thus ee have estimated the unknown coefficients $\mathbf{a}$ by the least square method[2].

The unknown coefficients $a_0, \dots a_9$ of the polynomial (2.3.1) are given by

$$\mathbf{a} = \mathbf{B_w f}, \tag{7}$$

where $\mathbf{B_w}$ is computed by formula

$$\mathbf{B_w} = inv(\mathbf{Q^T W^T W Q})\mathbf{Q^T W^T W}. \tag{8}$$

Size of matrix **Q** is 25×10, size of **a** is 10×1 (vector of unknown coefficients) and $\mathbf{f}$ is 25×1(vector of the nods).

The computation made in this way is very fast. The matrix $\mathbf{B_w}$ is computed only once during the first computation. We do not have to compute all the coefficients of $\mathbf{a}$, but only those we need for computation of the partial derivatives of the desired order. The matrix $\mathbf{B_w}$ was computed analytically (with the help of symbolic computations in Matlab). This helped to avoid the rounding error during computation of matrix $\mathbf{B_w}$, which made the computation even more precise.

Method presented by Florinski

[2] For the whole derivation of the weighted least square method see Pacina (2008).

Florinski (2009) introduced similar method for approximations of partial derivatives of the 3rd order based on Taylor approximating polynomial and standard least square method (without weights). The partial derivatives of the 3rd order along Florinski (2009) are computed in the following manner:

$$\frac{\partial^3 z}{\partial x^3} = \frac{z_5 + z_{10} + z_{15} + z_{20} + z_{25} - z_1 - z_6 - z_{11} - z_{16} - z_{21}}{10w^3} + \\ + \frac{2(z_2 + z_7 + z_{12} + z_{17} + z_{22} - z_4 - z_9 - z_{14} - z_{19} - z_{24})}{10w^3}, \qquad (9)$$

where z_i are the values from 5x5 neighborhood and w is the cell size.

Testing the methods

We have performed the accuracy test of the numerical evaluation of the derivatives on a test function with the character of topographical surface. The test function is given by mathematical formula which allows evaluation of all required derivatives and morphometrical variables analytically. Suitable test function *F(x,y)* given on a test area $A_F \subset R^2$ has to keep these properties:

- The function itself and all its partial derivatives of the first, second and third order are continuous functions in A_F .
- It is possible to express analytically all derivatives and all required morphometrical variables for any point in A_F.
- *F(x,y)* contains at least one saddle point and one peak in A_F.

We have used a polynomial function *P(x,y)* that is described in the following section as a test function

Polynomial testing function P(x,y)

We have used for testing the same function as used in Benová (2005) or Pacina (2009). This function *z = P(x,y)* is given on rectangular area A_P = (-300<x<300)×(-200<y<600) by formula

$$z = 150 + 0.2y - 1.5 \cdot 10^{-4}y^2 - 2 \cdot 10^{-7}y^3 + 0.1x + \\ +1.6 \cdot 10^{-4}xy - 1.2 \cdot 10^{-6}xy^2 + 10^{-4}x^2 + \\ +3.2 \cdot 10^{-6}x^2y + 2 \cdot 10^{-12}x^2y^3 - 10^{-6}x^3 - 10^{-12}x^3y^2 - \\ -10^{-14}x^3y^3 + 2.5 \cdot 10^{-17}x^3y^4 - 5 \cdot 10^{-17}x^4y^3 - 10^{-19}x^4y^4. \qquad (1.10)$$

The function is shown at figure Fig. 4. Such function is very smooth when generated in a raster with resolution 1 (i.e. raster of 600x800 cells), so the derivatives were computed very accurately even without weights. To get better insight into the influence of weights we have generated a raster Q from the same function at the same extent, but with resolution 50 (i.e. raster with only 12x16 cells).

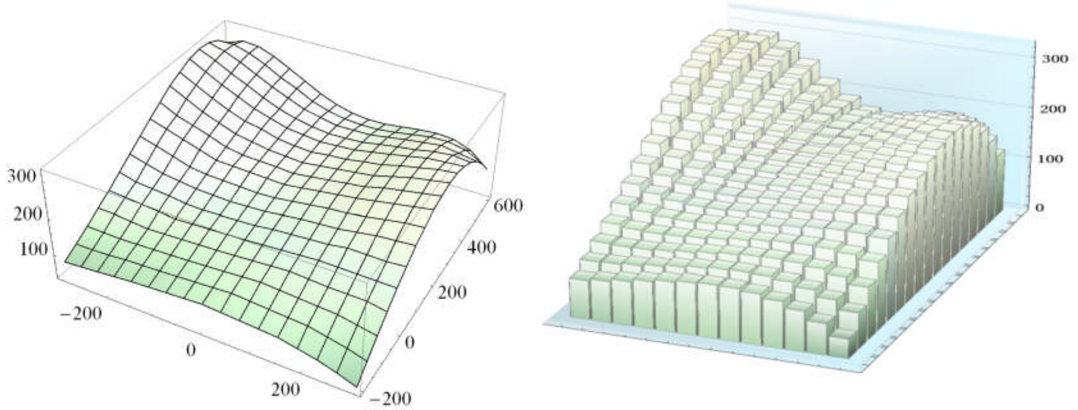

Fig. 4. Polynomial function P(x,y) plotted as a smooth surface and as raster Q with resolution h=50.

Analytical derivatives of function *P(x,y)* are given by following expressions:

$$\partial P(x,y)/\partial x = x^3 (-2.\times10^{-16}-4.\times10^{-19} y) y^3-1.2\times10^{-6} (-362.94+y) (229.606 +y)+7.5\times10^{-17} x^2 (-626.507+y) (387.984 +y) (164559. -161.477 y+y^2)+x (0.0002 +6.4\times10^{-6} y+4.\times10^{-12} y^3) \quad (1.11)$$

$$\partial^2 P(x,y)/\partial x^2 = 0.0002 +6.4\times10^{-6} y+4.\times10^{-12} y^3+x^2 (-6.\times10^{-16}-1.2\times10^{-18} y) y^3+1.5\times10^{-16} x (-626.507+y) (387.984 +y) (164559. -161.477 y+y^2) \quad (1.12)$$

$$\partial^3 P(x,y)/\partial x^3 = -6.\times10^{-6}-6.\times10^{-12} y^2+(-6.\times10^{-14}-1.2\times10^{-15} x) y^3+(1.5\times10^{-16}-2.4\times10^{-18} x) y^4 \quad (1.13)$$

$$\partial P(x,y)/\partial y = x (0.00016 -2.4\times10^{-6} y)+x^4 (-1.5\times10^{-16}-4.\times10^{-19} y) y^2+1.\times10^{-16} x^3 (-356.155+y) y (56.1553 +y)-6.\times10^{-7} (-379.153+y) (879.153 +y)+x^2 (3.2\times10^{-6}+6.\times10^{-12} y^2) \quad (1.14)$$

$$\partial^2 P(x,y)/\partial y^2 = -0.0003-2.4\times10^{-6} x-1.2\times10^{-6} y+1.2\times10^{-11} x^2 y+x^4 (-3.\times10^{-16}-1.2\times10^{-18} y) y+3.\times10^{-16} x^3 (-229.099+y) (29.0994 +y) \quad (1.15)$$

$$\partial^3 P(x,y)/\partial y^3 = -1.2\times10^{-6}+1.2\times10^{-11} x^2+x^4 (-3.\times10^{-16}-2.4\times10^{-18} y)+x^3 (-6.\times10^{-14}+6.\times10^{-16} y) \quad (1.16)$$

$$\partial^2 P(x,y)/\partial xy = 0.00016 -2.4\times10^{-6}y+x^3 (-6.\times10^{-16}-1.6\times10^{-18} y) y^2+3.\times10^{-16} x^2 (-356.155+y) y (56.1553 +y)+x (6.4\times10^{-6}+1.2\times10^{-11} y^2) \quad (1.17)$$

$$\partial^3 P(x,y)/\partial x^2 y= 6.4\times10^{-6}+1.2\times10^{-11}y^2+x^2 (-1.8\times10^{-15}-4.8\times10^{-18} y) y^2+6.\times10^{-16} x (-356.155+y) y (56.1553 +y) \quad (1.18)$$

$$\partial^3 P(x,y)/\partial x y^2 = -2.4\times10^{-6}+2.4\times10^{-11}x y+x^3 (-1.2\times10^{-15}-4.8\times10^{-18} y) y+9.\times10^{-16} x^2 (-229.099+y) (29.0994 +y) \quad (1.19)$$

The workflow

We have analyzed the approximation error for both rasters *F(x,y)=P(x,y)* with resolution *h=1* and *F(x,y)=Q(x,y)* with resolution *h=50* in the following way:

1. Evaluation of raster F^A of function values *F(x,y)* at given area with resolution *h.*
2. Evaluation of rasters F^A_x, F^A_{xx}, F^A_{xxx} of analytical derivatives at given area with resolution *h* from analytical formulae $\frac{\partial F(x,y)}{\partial x}, \frac{\partial^2 F(x,y)}{\partial x^2}, \frac{\partial^3 F(x,y)}{\partial x^3}, \ldots$
3. Numerical evaluation of derivatives ${}^mF^N_x$, ${}^mF^N_{xx}$, ${}^mF^N_{xxx}$ from raster F^A by particular numerical method *m*.
4. Evaluation of raster of differences ${}^m\Delta F_x = F^A_x - {}^mF^N_x$ between raster of analytical derivatives F^A_x and raster of numerical approximation ${}^mF^N_x$ for the first derivative $\frac{\partial F(x,y)}{\partial x}$ and similar rasters for all the other derivatives. The differences show absolute precision of various derivatives and can be used for studying of the effect of various methods for particular derivative. However, the absolute value of various derivatives significantly differs, so this absolute quantity cannot show us which derivative is computed with higher relative accuracy with respect to the other derivatives.
5. Evaluation of raster of equivalence ratios ${}^m\Pi F_x = {}^mF^N_x / F^A_x$ between raster of analytical derivatives F^A_x and raster of numerical approximation ${}^mF^N_x$ for the first derivative $\frac{\partial F(x,y)}{\partial x}$ and similar rasters for all the other derivatives. This indicator is a relative quantity that allows comparing accuracy of all computed derivatives regardless their absolute value. The ideal value is 1. Because derivatives F^A_x can be zero or small numbers, only values bigger than a chosen threshold value T are taken into account to avoid zero or small number in denominator (in tables, these values are referred as "filtered equivalence rates"). This manipulation is used only in points, where F^A_x is small. For our purposes are more important parts, where are third derivatives high. We have chosen a very small threshold T = $1\text{x}10^{-15}$.
6. Evaluation of statistical properties (mean, standard deviation, min and max) for rasters ${}^m\Delta F_x$ and ${}^m\Pi F_x$ for all methods and derivatives.

We have tested 9 methods with the following codes 1-9:

- Method 1: least squares (LS) without weights
- Method 2-5: LS with weights w^δ, $\delta = \{10, 1, 0.1, 0.02\}$
- Method 6-9: LS with weights w^ε , $\varepsilon = \{10, 1, 0.1, 0.02\}$

RESULTS

The results for dense (h = 1) raster P(x,y) are shown in tables Tab. 2 and Tab. 3 and results for raster Q(x,y) with the same extent, but with resolution h = 50 are shown in tables Tab. 4 and Tab. 5. Tables show, that the mean equivalence ratio is almost 1 with only negligible error for raster P(x,y). Also for raster Q(x,y), which is much more coarse, are the mean equivalence ratios between 0,95-1,25.

The minimal and maximal values of ratios look like very big in some cases, e.g. the maximal equivalence ratio of 36,76 for value ${}^{1}\Pi F_{xyy}$ means relative error 3500 % These big ratios are caused by dividing small numbers and big relative error for derivatives that are almost zero cannot much influence evaluation of important morphometrical parameters, because the wrong values will be also almost zero in absolute value. To prove that the absolute values are not too big, see statistics of ${}^{1}\Delta F_{xyy}$, where minimal and maximal differences are quite close to mean value of differences ${}^{1}\Delta F_{xyy}$.

CONCLUSIONS

- We consider the mean equivalence ratio as the main indicator of the accuracy, because it is a relative quantity, that allows comparison of all derivatives and witch can detect bias in results. The overview of all mean equivalence ratios (over whole grids) for each method and each derivative is given in table Tab. 1 (in addition to detailed results in tables **Table** 2 - **Table** 5.which provide more details in-depth)
- Best method is method 9 (weight w_{ij}^{ε} with parameter ε=0.02) that gives best mean equivalence ratios which up to about 20 % better than non-weighted least squares. This method has also the biggest minimal ratio values , the smallest maximal ratios values and the smallest standard deviation of the ratios. All these parameters proof that this method is the best one.
- All numerical experiments provided in this article show that any weighted method is better than the method without any weights.
- Even though we have generated the raster Q(x,y) with really coarse grid, the agreement of numerical derivatives with etalon (analytical derivatives) is quite good. Generating a smoother raster P(x,y) resulted to even better agreement.
- The numerical results are in a good agreement with our hypothesis that the weight of closer points should be higher (even though detailed statistical test is yet not available). This method should work also for real terrains.

Table 1. Mean equivalence ratio for derivatives computed from coarse (h = 50) raster Q(x,y)

	1	2	3	4	5	6	7	8	9
dx	1,00252	1,00135	1,00125	1,00124	1,00124	1,00057	1,00128	1,00030	1,00030
dy	0,97096	0,97957	0,98078	0,98092	0,98094	0,98543	0,98924	0,98839	0,98839
dxx	1,01591	1,00482	1,00408	1,00405	1,00405	0,99934	0,99983	0,99787	0,99786
dyy	1,14498	1,07123	1,06534	1,06512	1,06511	1,04777	1,05226	1,03765	1,03762
dxy	0,97374	0,97640	0,97668	0,97671	0,97671	0,98444	0,98721	0,98658	0,98659
dxxx	0,95100	0,96804	0,96949	0,96965	0,96966	0,98195	0,98396	0,98677	0,98678
dyyy	1,25471	1,15078	1,14006	1,13886	1,13876	1,07449	1,06819	1,04560	1,04570
dxxy	0,98565	0,98712	0,98728	0,98730	0,98730	0,99169	0,99749	0,99354	0,99354
dxyy	1,22670	1,20169	1,19899	1,19869	1,19866	1,12276	1,08759	1,09227	1,09226

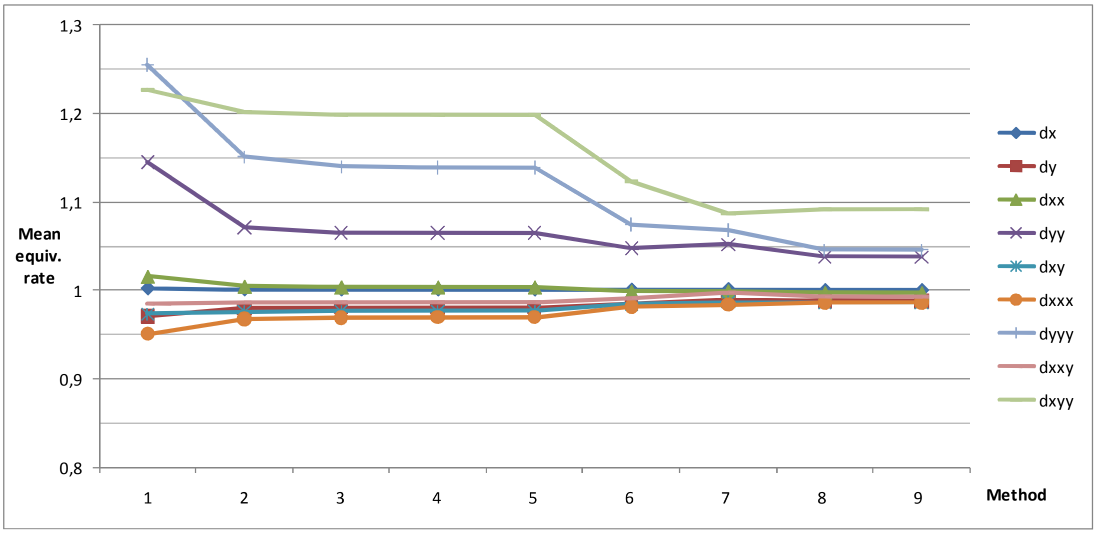

Fig. 5. Graph of mean equivalence ratios for raster Q(x,y) for various methods (weights).

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Table 2. Methods 1-5 for raster P with resolution h=1.

	Differences				*Filtered equivalence rates*			
	Method 1							
	$^{1}\Delta P_i$				$^{1}\Pi P_i$			
i	**mean**	**stdev**	**min**	**max**	**mean**	**stdev**	**min**	**max**
dx	5,1776E-11	6,0579E-10	-2,2383E-09	2,7376E-09	1,00000	0,00000	0,99974	1,00009
dy	-2,0533E-10	2,6783E-10	-1,6233E-09	2,9485E-10	1,00000	0,00000	0,99979	1,00168
dxx	9,1078E-08	1,4211E-07	-4,9185E-08	9,6619E-07	1,00000	0,01098	-3,89008	4,27598
dyy	5,7739E-08	1,0632E-07	-4,5983E-08	7,7106E-07	1,00003	0,00349	-0,10849	1,85696
dxy	-5,7089E-09	1,3218E-07	-7,4138E-07	5,8683E-07	0,99992	0,04100	-15,54950	6,27836
dxxx	-8,3282E-11	1,0600E-09	-4,7686E-09	3,9358E-09	1,00001	0,00092	0,91430	1,08996
dyyy	2,5400E-10	3,9467E-10	-5,3041E-10	2,3593E-09	1,00009	0,00034	0,97595	1,02308
dxxy	4,0827E-10	4,8268E-10	-2,7598E-10	2,5697E-09	1,00000	0,00103	0,91268	1,11756
dxyy	-6,6923E-11	6,2340E-10	-2,8549E-09	2,2650E-09	1,00002	0,00110	0,91702	1,09534
	Method 2							
	$^{2}\Delta P_i$				$^{2}\Pi P_i$			
dx	4,7629E-11	5,5465E-10	-2,0488E-09	2,5072E-09	1,00000	0,00000	0,99976	1,00008
dy	-1,9123E-10	2,4794E-10	-1,5010E-09	2,6951E-10	1,00000	0,00000	0,99981	1,00159
dxx	8,7040E-08	1,3514E-07	-4,6057E-08	9,1420E-07	1,00000	0,01043	-3,67754	4,09616
dyy	5,3930E-08	9,9309E-08	-4,2878E-08	7,2041E-07	1,00003	0,00329	-0,04250	1,80660
dxy	-5,6221E-09	1,3017E-07	-7,3012E-07	5,7791E-07	0,99992	0,04038	-15,29810	6,19819
dxxx	-7,7876E-11	9,9166E-10	-4,4610E-09	3,6821E-09	1,00001	0,00086	0,91985	1,08414
dyyy	2,3710E-10	3,6894E-10	-4,9627E-10	2,2048E-09	1,00009	0,00032	0,97750	1,02159
dxxy	4,0422E-10	4,7787E-10	-2,7073E-10	2,5389E-09	1,00000	0,00102	0,91318	1,11688
dxyy	-6,5969E-11	6,1253E-10	-2,8057E-09	2,2249E-09	1,00002	0,00109	0,91850	1,09364
	Method 3							
	$^{3}\Delta P_i$				$^{3}\Pi P_i$			
dx	3,5117E-11	4,0234E-10	-1,4847E-09	1,8204E-09	1,00000	0,00000	0,99983	1,00006
dy	-1,4689E-10	1,8699E-10	-1,1261E-09	1,9438E-10	1,00000	0,00000	0,99986	1,00127
dxx	7,1918E-08	1,0975E-07	-3,5019E-08	7,2572E-07	1,00000	0,00842	-2,87960	3,44730
dyy	3,9859E-08	7,4075E-08	-3,1940E-08	5,3809E-07	1,00002	0,00255	0,19401	1,62592
dxy	-5,3408E-09	1,2366E-07	-6,9358E-07	5,4900E-07	0,99992	0,03836	-14,48250	5,93807
dxxx	-6,1662E-11	7,8880E-10	-3,5476E-09	2,9296E-09	1,00001	0,00068	0,93643	1,06676
dyyy	1,8452E-10	2,9131E-10	-3,9533E-10	1,7351E-09	1,00007	0,00025	0,98206	1,01722
dxxy	3,9010E-10	4,6114E-10	-2,5337E-10	2,4335E-09	1,00000	0,00099	0,91504	1,11441
dxyy	-6,2746E-11	5,7636E-10	-2,6418E-09	2,0916E-09	1,00002	0,00102	0,92340	1,08800
	Method 4							
	$^{4}\Delta P_i$				$^{4}\Pi P_i$			
dx	2,7919E-11	3,1698E-10	-1,1690E-09	1,4348E-09	1,00000	0,00000	0,99986	1,00005
dy	-1,1912E-10	1,5037E-10	-9,0266E-10	1,5267E-10	1,00000	0,00000	0,99988	1,00105
dxx	5,8506E-08	8,8054E-08	-2,6083E-08	5,6641E-07	1,00000	0,00670	-2,16957	2,90291
dyy	2,7598E-08	5,3054E-08	-2,5843E-08	3,8551E-07	1,00001	0,00195	0,38119	1,47568
dxy	-5,1533E-09	1,1932E-07	-6,6923E-07	5,2972E-07	0,99993	0,03701	-13,93900	5,76470
dxxx	-5,2525E-11	6,7671E-10	-3,0423E-09	2,5143E-09	1,00001	0,00059	0,94572	1,05705
dyyy	1,5252E-10	2,4699E-10	-3,3992E-10	1,4620E-09	1,00005	0,00022	0,98455	1,01483
dxxy	3,7979E-10	4,4896E-10	-2,4201E-10	2,3592E-09	1,00000	0,00096	0,91659	1,11233
dxyy	-6,0551E-11	5,5241E-10	-2,5331E-09	2,0034E-09	1,00002	0,00098	0,92664	1,08428
	Method 5							
	$^{5}\Delta P_i$				$^{5}\Pi P_i$			
dx	2,6925E-11	3,0579E-10	-1,1277E-09	1,3842E-09	1,00000	0,00000	0,99987	1,00004
dy	-1,1522E-10	1,4535E-10	-8,7221E-10	1,4724E-10	1,00000	0,00000	0,99989	1,00102
dxx	5,7515E-08	8,6489E-08	-2,5477E-08	5,5518E-07	1,00000	0,00658	-2,11675	2,86464
dyy	2,6818E-08	5,1682E-08	-2,5953E-08	3,7552E-07	1,00001	0,00190	0,39418	1,46535
dxy	-5,1263E-09	1,1869E-07	-6,6573E-07	5,2695E-07	0,99993	0,03682	-13,86070	5,73974
dxxx	-5,1201E-11	6,6219E-10	-2,9766E-09	2,4607E-09	1,00001	0,00057	0,94695	1,05577
dyyy	1,4808E-10	2,4114E-10	-3,3280E-10	1,4254E-09	1,00005	0,00021	0,98486	1,01453
dxxy	3,7823E-10	4,4712E-10	-2,4040E-10	2,3482E-09	1,00000	0,00096	0,91684	1,11199
dxyy	-6,0227E-11	5,4898E-10	-2,5175E-09	1,9908E-09	1,00002	0,00097	0,92710	1,08375

Table 3. Methods 1 and 6-9 for raster P with resolution h=1.

	Differences				***Filtered equivalence rates***			
	Method 1							
	$^{1}\Delta P_i$				$^{1}\Pi P_i$			
i	**mean**	**stdev**	**min**	**max**	**mean**	**stdev**	**min**	**max**
dx	5,1776E-11	6,0579E-10	-2,2383E-09	2,7376E-09	1,00000	0,00000	0,99974	1,00009
dy	-2,0533E-10	2,6783E-10	-1,6233E-09	2,9485E-10	1,00000	0,00000	0,99979	1,00168
dxx	9,1078E-08	1,4211E-07	-4,9185E-08	9,6619E-07	1,00000	0,01098	-3,89008	4,27598
dyy	5,7739E-08	1,0632E-07	-4,5983E-08	7,7106E-07	1,00003	0,00349	-0,10849	1,85696
dxy	-5,7089E-09	1,3218E-07	-7,4138E-07	5,8683E-07	0,99992	0,04100	-15,54950	6,27836
dxxx	-8,3282E-11	1,0600E-09	-4,7686E-09	3,9358E-09	1,00001	0,00092	0,91430	1,08996
dyyy	2,5400E-10	3,9467E-10	-5,3041E-10	2,3593E-09	1,00009	0,00034	0,97595	1,02308
dxxy	4,0827E-10	4,8268E-10	-2,7598E-10	2,5697E-09	1,00000	0,00103	0,91268	1,11756
dxyy	-6,6923E-11	6,2340E-10	-2,8549E-09	2,2650E-09	1,00002	0,00110	0,91702	1,09534
	Method 6							
	$^{6}\Delta P_i$				$^{6}\Pi P_i$			
dx	4,7351E-11	5,5119E-10	-2,0360E-09	2,4916E-09	1,00000	0,00000	0,99976	1,00008
dy	-1,9032E-10	2,4665E-10	-1,4929E-09	2,6780E-10	1,00000	0,00000	0,99981	1,00158
dxx	8,6802E-08	1,3472E-07	-4,5856E-08	9,1098E-07	1,00000	0,01039	-3,66507	4,08494
dyy	5,3706E-08	9,8875E-08	-4,2677E-08	7,1727E-07	1,00003	0,00327	-0,03835	1,80343
dxy	-5,6063E-09	1,2981E-07	-7,2806E-07	5,7629E-07	0,99992	0,04026	-15,25220	6,18353
dxxx	-7,7498E-11	9,8674E-10	-4,4389E-09	3,6638E-09	1,00001	0,00086	0,92024	1,08373
dyyy	2,3604E-10	3,6717E-10	-4,9380E-10	2,1944E-09	1,00009	0,00032	0,97761	1,02149
dxxy	4,0373E-10	4,7728E-10	-2,7010E-10	2,5352E-09	1,00000	0,00102	0,91324	1,11680
dxyy	-6,5853E-11	6,1122E-10	-2,7998E-09	2,2201E-09	1,00002	0,00108	0,91867	1,09344
	Method 7							
	$^{7}\Delta P_i$				$^{7}\Pi P_i$			
dx	8,3848E-12	8,1531E-11	-2,9741E-10	3,7231E-10	1,00000	0,00000	0,99997	1,00001
dy	-4,7564E-11	5,6369E-11	-3,0941E-10	3,6907E-11	1,00000	0,00000	0,99996	1,00052
dxx	4,5393E-08	6,6757E-08	-1,5219E-08	3,9173E-07	1,00000	0,00493	-1,47895	2,29338
dyy	1,6496E-08	3,0465E-08	-1,2444E-08	2,2260E-07	1,00001	0,00126	0,59108	1,30567
dxy	-3,1776E-09	7,3574E-08	-4,1267E-07	3,2664E-07	0,99996	0,02282	-8,21174	3,93802
dxxx	-2,2389E-11	2,9445E-10	-1,3220E-09	1,0954E-09	1,00000	0,00025	0,97676	1,02449
dyyy	5,7459E-11	1,0370E-10	-1,4912E-10	5,9540E-10	1,00002	0,00010	0,99318	1,00655
dxxy	2,6595E-10	3,1805E-10	-1,0390E-10	1,5130E-09	1,00000	0,00072	0,93187	1,09197
dxyy	-3,4809E-11	2,6450E-10	-1,2268E-09	9,4102E-10	1,00001	0,00048	0,96570	1,03938
	Method 8							
	$^{8}\Delta P_i$				$^{8}\Pi P_i$			
dx	8,3848E-12	8,1531E-11	-2,9741E-10	3,7231E-10	1,00000	0,00000	0,99997	1,00001
dy	-4,7564E-11	5,6369E-11	-3,0941E-10	3,6907E-11	1,00000	0,00000	0,99996	1,00052
dxx	4,5393E-08	6,6757E-08	-1,5219E-08	3,9173E-07	1,00000	0,00493	-1,47895	2,29338
dyy	1,6496E-08	3,0465E-08	-1,2444E-08	2,2260E-07	1,00001	0,00126	0,59108	1,30567
dxy	-3,1776E-09	7,3574E-08	-4,1267E-07	3,2664E-07	0,99996	0,02282	-8,21174	3,93802
dxxx	-2,2389E-11	2,9445E-10	-1,3220E-09	1,0954E-09	1,00000	0,00025	0,97676	1,02449
dyyy	5,7459E-11	1,0370E-10	-1,4912E-10	5,9540E-10	1,00002	0,00010	0,99318	1,00655
dxxy	2,6595E-10	3,1805E-10	-1,0390E-10	1,5130E-09	1,00000	0,00072	0,93187	1,09197
dxyy	-3,4809E-11	2,6450E-10	-1,2268E-09	9,4102E-10	1,00001	0,00048	0,96570	1,03938
	Method 9							
	$^{9}\Delta P_i$				$^{9}\Pi P_i$			
dx	8,3848E-12	8,1531E-11	-2,9741E-10	3,7231E-10	1,00000	0,00000	0,99997	1,00001
dy	-4,7564E-11	5,6369E-11	-3,0941E-10	3,6907E-11	1,00000	0,00000	0,99996	1,00052
dxx	4,5393E-08	6,6757E-08	-1,5219E-08	3,9173E-07	1,00000	0,00493	-1,47895	2,29338
dyy	1,6496E-08	3,0465E-08	-1,2444E-08	2,2260E-07	1,00001	0,00126	0,59108	1,30567
dxy	-3,1776E-09	7,3574E-08	-4,1267E-07	3,2664E-07	0,99996	0,02282	-8,21174	3,93802
dxxx	-2,2389E-11	2,9445E-10	-1,3220E-09	1,0954E-09	1,00000	0,00025	0,97676	1,02449
dyyy	5,7459E-11	1,0370E-10	-1,4912E-10	5,9540E-10	1,00002	0,00010	0,99318	1,00655
dxxy	2,6595E-10	3,1805E-10	-1,0390E-10	1,5130E-09	1,00000	0,00072	0,93187	1,09197
dxyy	-3,4809E-11	2,6450E-10	-1,2268E-09	9,4102E-10	1,00001	0,00048	0,96570	1,03938

Table 4. Methods 1-5 for raster Q with resolution h=50.

	Differences				***Filtered equivalence rates***			
Method 1								
	$^{1}\Delta Q_i$				$^{1}\Pi Q_i$			
i	mean	stdev	min	max	mean	stdev	min	max
dx	3,8551E-04	4,3954E-03	-1,4215E-02	1,7387E-02	1,00252	0,06964	0,24013	1,49803
dy	-1,5556E-03	2,0247E-03	-1,0425E-02	1,8991E-03	0,97096	0,58189	-7,49817	2,37510
dxx	2,8727E-04	4,4579E-04	-1,2261E-04	2,4980E-03	1,01591	0,53611	-2,10224	5,30039
dyy	1,9191E-04	3,4005E-04	-1,1334E-04	1,9989E-03	1,14498	0,88616	-6,74476	9,70580
dxy	-1,8148E-05	4,0980E-04	-1,9132E-03	1,5133E-03	0,97374	1,00615	-11,70870	4,23124
dxxx	-2,4638E-07	3,0701E-06	-1,2103E-05	9,9902E-06	0,95100	1,88624	-25,61410	8,57538
dyyy	7,9215E-07	1,2016E-06	-1,3637E-06	6,0620E-06	1,25471	0,36751	-1,56791	3,39896
dxxy	1,1764E-06	1,4036E-06	-7,1205E-07	6,5566E-06	0,98565	0,81374	-4,33275	9,61478
dxyy	-1,9883E-07	1,8141E-06	-7,2533E-06	5,7550E-06	1,22670	2,65098	-5,07256	36,76390
Method 2								
	$^{2}\Delta Q_i$				$^{2}\Pi Q_i$			
dx	2,1559E-04	2,3933E-03	-7,7252E-03	9,4804E-03	1,00135	0,03788	0,58740	1,27122
dy	-9,2505E-04	1,1729E-03	-6,0091E-03	1,0234E-03	0,97957	0,39209	-4,73816	1,88628
dxx	1,8596E-04	2,8089E-04	-6,8183E-05	1,5118E-03	1,00482	0,33660	-1,11774	3,49531
dyy	9,7091E-05	1,7781E-04	-5,9621E-05	1,0463E-03	1,07123	0,50327	-3,43836	5,87902
dxy	-1,6427E-05	3,7152E-04	-1,7352E-03	1,3725E-03	0,97640	0,91128	-10,50950	3,92711
dxxx	-1,5876E-07	2,0070E-06	-7,9094E-06	6,5371E-06	0,96804	1,23270	-16,40190	5,92395
dyyy	4,9122E-07	7,7140E-07	-8,9437E-07	3,8581E-06	1,15078	0,23808	-0,68742	2,57851
dxxy	1,0959E-06	1,3078E-06	-6,2904E-07	6,0467E-06	0,98712	0,77237	-4,10113	9,24444
dxyy	-1,8078E-07	1,6188E-06	-6,4792E-06	5,1257E-06	1,20169	2,37231	-4,46470	32,84020
Method 3								
	$^{3}\Delta Q_i$				$^{3}\Pi Q_i$			
dx	2,0019E-04	2,2187E-03	-7,1609E-03	8,7899E-03	1,00125	0,03511	0,61765	1,25144
dy	-8,6228E-04	1,0918E-03	-5,5912E-03	9,4813E-04	0,98078	0,36804	-4,38672	1,83002
dxx	1,7710E-04	2,6699E-04	-6,3895E-05	1,4299E-03	1,00408	0,31978	-1,02746	3,34949
dyy	8,9489E-05	1,6505E-04	-5,5462E-05	9,7092E-04	1,06534	0,47259	-3,17182	5,57330
dxy	-1,6241E-05	3,6736E-04	-1,7158E-03	1,3572E-03	0,97668	0,90100	-10,37970	3,89417
dxxx	-1,5105E-07	1,9163E-06	-7,5512E-06	6,2432E-06	0,96949	1,17704	-15,61830	5,69498
dyyy	4,6240E-07	7,3313E-07	-8,5475E-07	3,6569E-06	1,14006	0,22721	-0,61376	2,50952
dxxy	1,0861E-06	1,2962E-06	-6,2019E-07	5,9871E-06	0,98728	0,76686	-4,06835	9,19182
dxyy	-1,7875E-07	1,5977E-06	-6,3951E-06	5,0578E-06	1,19899	2,34199	-4,39776	32,41670
Method 4								
	$^{4}\Delta Q_i$				$^{4}\Pi Q_i$			
dx	1,9849E-04	2,1996E-03	-7,0990E-03	8,7141E-03	1,00124	0,03480	0,62096	1,24927
dy	-8,5527E-04	1,0828E-03	-5,5449E-03	9,3988E-04	0,98092	0,36527	-4,34635	1,82362
dxx	1,7666E-04	2,6630E-04	-6,3698E-05	1,4260E-03	1,00405	0,31895	-1,02274	3,34265
dyy	8,9198E-05	1,6452E-04	-5,5281E-05	9,6781E-04	1,06512	0,47119	-3,15954	5,55946
dxy	-1,6220E-05	3,6689E-04	-1,7136E-03	1,3555E-03	0,97671	0,89984	-10,36500	3,89045
dxxx	-1,5021E-07	1,9064E-06	-7,5120E-06	6,2111E-06	0,96965	1,17096	-15,53270	5,66992
dyyy	4,5921E-07	7,2892E-07	-8,5043E-07	3,6346E-06	1,13886	0,22603	-0,60574	2,50200
dxxy	1,0850E-06	1,2948E-06	-6,1919E-07	5,9803E-06	0,98730	0,76622	-4,06452	9,18568
dxyy	-1,7852E-07	1,5953E-06	-6,3857E-06	5,0501E-06	1,19869	2,33856	-4,39020	32,36890
Method 5								
	$^{5}\Delta Q_i$				$^{5}\Pi Q_i$			
dx	1,9834E-04	2,1979E-03	-7,0935E-03	8,7073E-03	1,00124	0,03478	0,62126	1,24908
dy	-8,5464E-04	1,0820E-03	-5,5407E-03	9,3914E-04	0,98094	0,36503	-4,34271	1,82304
dxx	1,7663E-04	2,6625E-04	-6,3684E-05	1,4257E-03	1,00405	0,31889	-1,02237	3,34215
dyy	8,9179E-05	1,6448E-04	-5,5268E-05	9,6759E-04	1,06511	0,47108	-3,15863	5,55845
dxy	-1,6219E-05	3,6685E-04	-1,7134E-03	1,3554E-03	0,97671	0,89974	-10,36370	3,89012
dxxx	-1,5013E-07	1,9055E-06	-7,5085E-06	6,2082E-06	0,96966	1,17041	-15,52500	5,66767
dyyy	4,5892E-07	7,2855E-07	-8,5004E-07	3,6327E-06	1,13876	0,22593	-0,60502	2,50132
dxxy	1,0849E-06	1,2947E-06	-6,1911E-07	5,9797E-06	0,98730	0,76617	-4,06418	9,18513
dxyy	-1,7850E-07	1,5951E-06	-6,3848E-06	5,0494E-06	1,19866	2,33826	-4,38952	32,36470

Table 5. Methods 1 and 6-9 for raster Q with resolution h=50.

	Differences				***Filtered equivalence rates***			
				Method 1				
	$^{1}\Delta Q_i$				$^{1}\Pi Q_i$			
i	**mean**	**stdev**	**min**	**max**	**mean**	**stdev**	**min**	**max**
dx	3,8551E-04	4,3954E-03	-1,4215E-02	1,7387E-02	1,00252	0,06964	0,24013	1,49803
dy	-1,5556E-03	2,0247E-03	-1,0425E-02	1,8991E-03	0,97096	0,58189	-7,49817	2,37510
dxx	2,8727E-04	4,4579E-04	-1,2261E-04	2,4980E-03	1,01591	0,53611	-2,10224	5,30039
dyy	1,9191E-04	3,4005E-04	-1,1334E-04	1,9989E-03	1,14498	0,88616	-6,74476	9,70580
dxy	-1,8148E-05	4,0980E-04	-1,9132E-03	1,5133E-03	0,97374	1,00615	-11,70870	4,23124
dxxx	-2,4638E-07	3,0701E-06	-1,2103E-05	9,9902E-06	0,95100	1,88624	-25,61410	8,57538
dyyy	7,9215E-07	1,2016E-06	-1,3637E-06	6,0620E-06	1,25471	0,36751	-1,56791	3,39896
dxxy	1,1764E-06	1,4036E-06	-7,1205E-07	6,5566E-06	0,98565	0,81374	-4,33275	9,61478
dxyy	-1,9883E-07	1,8141E-06	-7,2533E-06	5,7550E-06	1,22670	2,65098	-5,07256	36,76390
				Method 6				
	$^{6}\Delta Q_i$				$^{6}\Pi Q_i$			
dx	1,0065E-04	1,0202E-03	-3,2422E-03	4,0364E-03	1,00057	0,01608	0,82624	1,11548
dy	-4,9575E-04	5,9731E-04	-2,9698E-03	4,1509E-04	0,98543	0,26305	-2,86052	1,55329
dxx	1,5145E-04	2,2506E-04	-4,6215E-05	1,1480E-03	0,99934	0,26766	-0,80726	2,79531
dyy	6,6400E-05	1,1876E-04	-3,8129E-05	7,0206E-04	1,04777	0,35664	-2,17464	4,41388
dxy	-1,1233E-05	2,5598E-04	-1,1979E-03	9,4787E-04	0,98444	0,62489	-6,88926	3,00910
dxxx	-8,8413E-08	1,1385E-06	-4,4834E-06	3,7098E-06	0,98195	0,69881	-8,87098	3,77339
dyyy	2,6010E-07	4,2872E-07	-5,0816E-07	2,1169E-06	1,07449	0,13531	0,04104	1,89952
dxxy	8,5216E-07	1,0236E-06	-3,6571E-07	4,4818E-06	0,99169	0,65415	-3,44538	8,19794
dxyy	-1,2471E-07	1,0039E-06	-4,0395E-06	3,1387E-06	1,12276	1,50196	-3,15264	20,46080
				Method 7				
	$^{7}\Delta Q_i$				$^{7}\Pi Q_i$			
dx	1,5182E-04	7,2910E-04	-1,1112E-03	2,0704E-03	1,00128	0,00938	0,89925	1,06705
dy	-4,7888E-04	2,9674E-04	-1,3929E-03	-1,1625E-04	0,98924	0,20599	-2,02586	1,41784
dxx	1,4264E-04	2,0450E-04	-3,5272E-05	1,0286E-03	0,99983	0,25289	-0,71687	2,70784
dyy	6,0891E-05	9,8556E-05	-2,9210E-05	5,8644E-04	1,05226	0,37106	-2,10374	4,58541
dxy	-9,7200E-06	2,2676E-04	-1,0666E-03	8,4416E-04	0,98721	0,55010	-5,94354	2,76991
dxxx	-9,4640E-08	8,9681E-07	-3,1151E-06	2,3306E-06	0,98396	0,62315	-7,73297	3,67021
dyyy	2,2727E-07	3,0314E-07	-2,8017E-07	1,2568E-06	1,06819	0,10064	0,58393	1,31323
dxxy	7,4089E-07	9,4023E-07	-3,0814E-07	4,0411E-06	0,99749	0,60204	-3,15404	7,73444
dxyy	-9,3041E-08	7,7691E-07	-3,2713E-06	2,5760E-06	1,08759	1,11411	-2,94535	15,06500
				Method 8				
	$^{8}\Delta Q_i$				$^{8}\Pi Q_i$			
dx	6,0082E-05	5,7789E-04	-1,8433E-03	2,3078E-03	1,00030	0,00910	0,90296	1,06553
dy	-3,3608E-04	4,0422E-04	-1,9353E-03	2,3135E-04	0,98839	0,20367	-1,99257	1,41370
dxx	1,3508E-04	2,0025E-04	-3,8110E-05	9,9550E-04	0,99787	0,23725	-0,64957	2,52023
dyy	5,3255E-05	9,5698E-05	-3,0369E-05	5,6631E-04	1,03765	0,29921	-1,67548	3,84190
dxy	-9,7938E-06	2,2368E-04	-1,0474E-03	8,2883E-04	0,98658	0,54524	-5,88289	2,75354
dxxx	-6,3595E-08	8,3794E-07	-3,3003E-06	2,7367E-06	0,98677	0,51367	-6,26115	3,02225
dyyy	1,7591E-07	3,0939E-07	-3,7627E-07	1,5002E-06	1,04560	0,10222	0,28792	1,67123
dxxy	7,4333E-07	8,9943E-07	-2,6587E-07	3,8155E-06	0,99354	0,59504	-3,08714	7,62325
dxyy	-1,0180E-07	7,6544E-07	-3,0921E-06	2,3727E-06	1,09227	1,16564	-2,83711	15,67250
				Method 9				
	$^{9}\Delta Q_i$				$^{9}\Pi Q_i$			
dx	6,0026E-05	5,7723E-04	-1,8412E-03	2,3052E-03	1,00030	0,00909	0,90307	1,06546
dy	-3,3587E-04	4,0396E-04	-1,9338E-03	2,3106E-04	0,98839	0,20360	-1,99155	1,41352
dxx	1,3501E-04	2,0015E-04	-3,8084E-05	9,9494E-04	0,99786	0,23713	-0,64880	2,51929
dyy	5,3219E-05	9,5632E-05	-3,0347E-05	5,6592E-04	1,03762	0,29902	-1,67382	3,84008
dxy	-9,7876E-06	2,2354E-04	-1,0467E-03	8,2831E-04	0,98659	0,54490	-5,87858	2,75244
dxxx	-6,3549E-08	8,3719E-07	-3,2974E-06	2,7342E-06	0,98678	0,51321	-6,25455	3,02056
dyyy	1,7590E-07	3,0917E-07	-3,7591E-07	1,4994E-06	1,04570	0,10210	0,28862	1,67059
dxxy	7,4307E-07	8,9910E-07	-2,6586E-07	3,8143E-06	0,99354	0,59479	-3,08537	7,62037
dxyy	-1,0178E-07	7,6534E-07	-3,0917E-06	2,3724E-06	1,09226	1,16544	-2,83533	15,67080