PERSPECTIVES OF FRACTAL GEOMETRY IN GIS ANALYSES

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Abstract

Together with rapid development in GI science recent decades, the fractal geometry represents a powerful tool for various geographic analyses and studies. The paper shows usage of fractal geometry in two case studies. Studied areas are Olomouc region (approx. 804 km^2) and Olomouc city itself (100.000 inhabitants). First case study concerns urban growth of Olomouc city and refers about relationship between its area and perimeter. Fractal analyses showed that Olomouc is now approximately in the middle of its growth process and especially inner parts of the city are sufficient to be developed. Second case study pointed the land cover areas with extreme values of fractal dimension in Olomouc region. This led, together with consequent statistical analyses, to result that according to fractal dimension it is possible to distinguish (or at least to assume) the origin of areas. To achieve the results, various methods were employed. For fractal dimension calculation in the first case study, the box-counting method was used. General fractal calculation method was used in the second case study. Some statistical methods were also applied to test mean values of land cover areas fractal dimension (Student's t-test and analysis of variance). Using non-integer, fractal dimension, one can analyze complexity of the shape, explore underlying geographic processes and analyze various geographic phenomena in a new and innovative way.

Keywords: **fractal geometry, GIS, urban growth, land cover, geocomputation, box-counting dimension, area/perimeter relation**

1. INTRODUCTION

When Weierstrass's continuous nonwhere-differentiable curve appeared in 1875, it was called by other mathematicians as "regrettable evil" and these types of object were known as mathematical "monsters" [8, 17]. Nobody imagined that fundamentals of fractal geometry were just established. However, since Mandelbrot's published its basics in [12], fractal geometry and fractal dimension (non-integer dimension, e.g. 1.32 D) is well known as a valuable tool for describing the shape of objects. It gained large popularity in geosciences [1, 7] (among other disciplines), where the measures of object's shape are essential. Year 1994 might be considered as the beginning of exploration of cities by means of fractal geometry [1]. Number of ways in which fractal geometry (and especially fractal dimension) can be used for examining the form of city and this paper's first case study follows their work in a certain way.

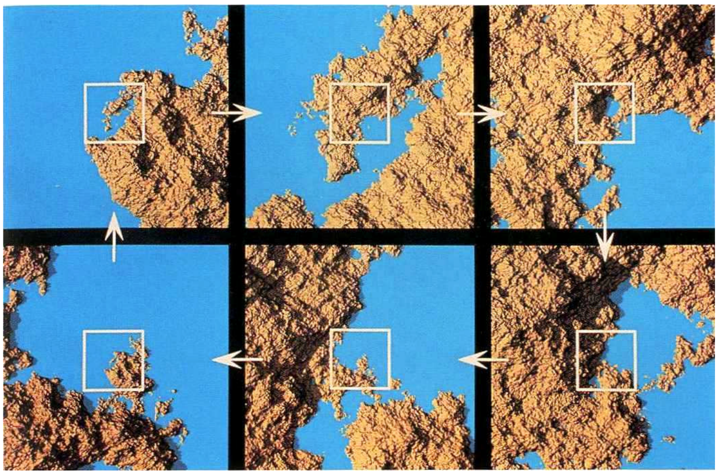

Fig. 1. Example of fractal coast and scale-invariance principle (in six steps/scales) [17].

More complex and detailed information about fractal geometry is in [8, 11, 13, 14, 17, 18]. Books provide the broad view of the underlying notions behind fractals and, in addition, show how fractals and chaos theory relate to each other as well as to natural phenomena. Especially introduction of fractals to the reader with the explicit link to natural sciences, such as ecology, geography (demography), physical geography, spatio-temporal analyses and others is in [8]. Some papers concerning topics investigated in this paper (city growth and land use pattern) were published yet, e.g. Batty and Longley's book [1] as pioneer work. Other studies, such as [2, 5, 6, 15, 22, 25], applied different fractal methods for description of city morphology. Fractal analyses applied on land use/land cover pattern are described as well, such as in [5, 9, 10, 16, 20, 26, 27].

One of the major principles in chaos theory and descriptive fractal geometry is self-similarity and self-affinity. The most theoretical fractal objects, such as Mandelbrot set, are self-similar – this means that any part of the object is exactly similar to the whole. But these types of fractals are rarely used to approximate objects or shapes from the real world. And thus, another type of fractals is suitable for real-world object description – self-affine ones. These fractals are in fact self-similar too, but transformed via affine transformation (e.g. translation, rotation, scaling, shear mapping) of the whole or the part of fractal object [1, 8, 11, 14, 17, 18]. This observation is closely related to scale-invariance, which means that object has same properties in any scale, in any detail. In other words, if characteristics of some fractal object are known in certain scale, it is possible to anticipate these characteristics of another fractal object in different scale. The very typical example of this object is land cover and/or urban forms with theirs dynamics.

Concept of fractality was described in detail in many publications [4, 7, 14, 17, 21]. Fractal dimension is a measure of complexity of shape, based on irregularity, scale dependency and self-similarily of objects [2]. The basic property of all fractal structures is their dimension. Although there is no exact definition of fractals, the publicly accepted one, coming from Mandelbrot himself: "A fractal is by definition a set for which the Hausdorff-Besicovitch dimension strictly exceeds the topological dimension" [14]. Hausdorff-Besicovith dimension is therefore a number, which describes the complexity of an object and its value is non-integer. The bigger the value of Hausdorff-Besicovitch dimension, the more complex the shape of object and the more fills the space. In sense of Euclidean geometry, dimension is 1 for straight line, 2 for circle or square and 3 for cube or sphere, all. For real objects in plane, Hausdorff-Besicovitch dimension (fractal dimension) has values greater than 1 and less than 2. It obvious that Euclidean, integer, dimension is extreme case of fractal, non-integer, dimension. So it is claimed that regions with regular and less complex shape has lower fractal dimension (approaching to 1) and vice versa – the more irregular and complex shapes, the higher fractal dimension (approaching to 2). Values of fractal dimension of land cover regions vary between 1 and 2 because of the fact that area represented in the plane space without vertical extend is in fact enclosed curves. And fractal dimension of curves lies between 1 and 2.

As depicted in mentioned publications, fractals provide tool for better understanding the shape of given object. Furthermore, fractal geometry brings very effective apparatus to measure object's dimension and shape metrics in order to supply or even substitute other measurable characteristics of the object. Fractal dimension value is independent on area or perimeter of the object. Two objects with same area or perimeter could have absolutely different fractal dimensions. This theoretical notion is very important point, because as shown further, there is a relation between area occupied by the city and its fractal dimension. As [1] shows, fractals have infinite perimeter and finite area. Therefore it seems pointless to explore the perimeter of the city by means of fractal dimension. However, finding relation between area, perimeter, fractal dimension of build-up areas and fractal dimension of the boundary of the city is the scope of the analysis.

Next paragraphs do not intent to completely identify socio-economical, demographical and geographical aspects of land cover current state in Olomouc region. The case studies demonstrate the opportunity and power of fractal analyses of geographical data. Particularly, objectives are: urban structure of the city (shape of its borders) and land cover pattern and its geometric representation in GIS. In the first case, a possible explanation of city growth due to fractal analyses of its borders is stated. Land cover pattern fractal analyses, among others, identify areas with maximal and minimal fractal dimension to evaluate complexity of such areas.

2. DATA AND METHODS

There is a number of methods for estimating fractal dimension and as [19] shows, results obtained by different methods often differ significantly. Also not only the method itself, but also the software, which calculates the fractal dimension may contribute to the differences [19]. In this case, Fractalyse software is used [24].

It has to be mentioned that in case study 2 statistical testing was used. Because of its well-known formulas and characteristics, detailed description is not stated. The methods were Shapiro-Wilk test of normal distribution, Student's t-test and analysis of variance (hereafter as ANOVA).

2.1 Box-counting method

The box-counting method was used for modified data – binary pictures. Box-counting dimension of a subset *X* of the plain is defined by counting number of unit boxes which intersects *X*: for any $\Delta s > 0$, let $N(\Delta s)$ denote the minimum number of *n*-dimensional cubes of linear scale Δs (side length) needed to cover *X*. Than *X* has box dimension *D* if $N(\Delta s)$ satisfies (according to [8, 23]):

$$N(\Delta s) \approx c(1/\Delta s)^D, \qquad (1)$$

where $\Delta s \to 0$, *c* is a constant and box-counting dimension of *X* is *D*. Formula (1) is called power law.

Dimension *D* is then be computed by:

$$D = \lim_{\Delta s \to 0}\left[-\log N(\Delta s)/\log \Delta s\right], \qquad (2)$$

According to formula (2), calculation of box-counting dimension is simple. For a sequence of cell size $\Delta s > 0$, the number of cells $N(\Delta s)$ needed to cover the set *S* is calculated. Box-counting dimension *D* can be also estimated by the slope of the straight line formed by plotting $\log N\Delta s$ against $\log \Delta s$ (known also as Richardson-Mandelbrot plot [3]). If the trend is linear, one can assume the observed object to be fractal [6, 8, 17].

2.2 General fractal calculation method

The same principles as in previous method employ method for calculating fractal dimension in its most general form:

$$D = \frac{2 \cdot \log P}{\log A}, \quad (3)$$

where *P* is the perimeter of the space being studied at a particular length scale, and *A* is the two-dimensional area of the space under investigation [25]. Formula (3) was used to calculate fractal dimension for land cover regions classified by Level 1 of the hierarchy. Formula (3) can be easily computable directly within GIS, thus ESRI ArcGIS 9.x was used in order to obtain fractal dimension values.

2.3 Study region

Olomouc city is typical in many aspects. Having its center on small hill above the river Morava and surrounded by flat land, it does not have natural boundaries at all and urban growth is therefore not limited by georelief. The case study uses aerial imagery from years 1926, 1971, 1978, 1991, 2001, 2003 and 2006 to identify the boundary of the city. Both the central city and its surroundings were considered as study area, since the urbanization process of Olomouc city was heavily influenced by its surroundings. From Fig. 2, it is clear to see, that the building-up was realized primary in directions of former surrounding villages, therefore it was necessary to include those into calculation in case study 1.

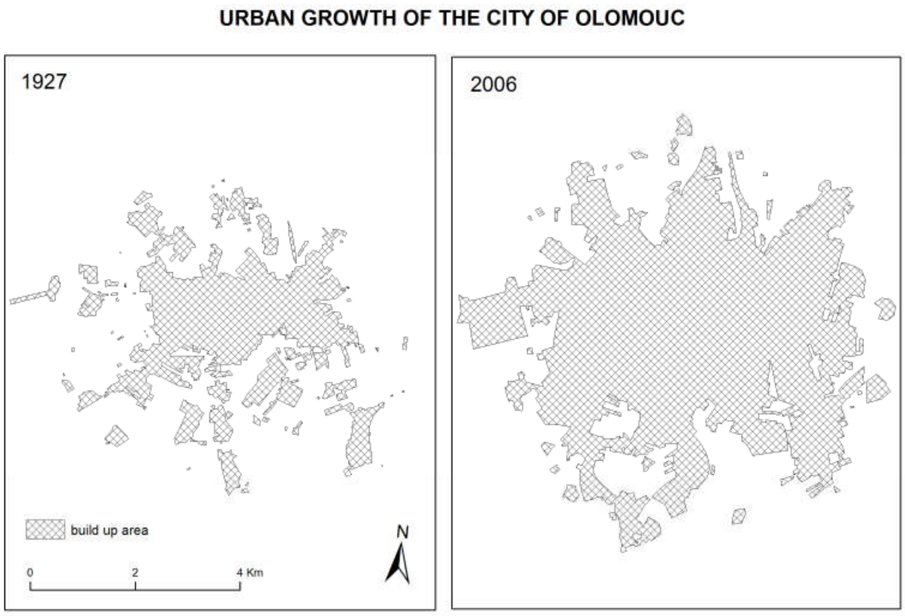

Fig. 2. Urban form of Olomouc in 1927 (left) and 2006 (right).

For case study 2, territory of Olomouc region was used. Its area is approximately 804 km^2 and every single type of LEVEL1 land cover classification is represented. It is necessary to note that CORINE Land Cover dataset from year 2000 was examined. Olomouc region is mainly covered by the agricultural areas, but the north-east part is almost completely covered by forests, because of military area occurrence. Despite this fact, Olomouc region is the most typically agricultural region with a great number of dispersed villages.

3. CASE STUDY 1 – FRACTAL ANALYSIS OF URBAN GROWTH OF OLOMOUC CITY

Observation of values of fractal dimension of Olomouc as whole is extended by examining the relation between the fractal dimension and other descriptors of shape, namely area and perimeter, which later appeared to be very important. As mentioned above, fractal objects does not have finite perimeter but have finite area, so authors suppose, that there will be relation between fractal dimension and area, but none between fractal dimension and perimeter.

Measured values of fractal dimension of both build-up areas and their boundary are shown in Tab. 1.

Tab. 1. Descriptors of shape of Olomouc

Year	Area (km^2)	D_{Area}	Perimeter (km)	$D_{Perimeter}$
1927	10.485	1.683	106.57	1.363
1971	21.188	1.758	116.61	1.284
1978	23.780	1.780	117.14	1.285
1991	28.028	1.808	108.12	1.268
2001	28.726	1.813	112.29	1.274
2003	29.064	1.816	111.28	1.272
2006	29.639	1.816	112.36	1.272

It is obvious, that both area and its fractal dimension grow constantly in time. Perimeter, and its fractal dimension, on the other hand, does not seem to have a strict trend. This would only support out hypothesis presented in previous text. Plotted values are shown in Fig. 3.

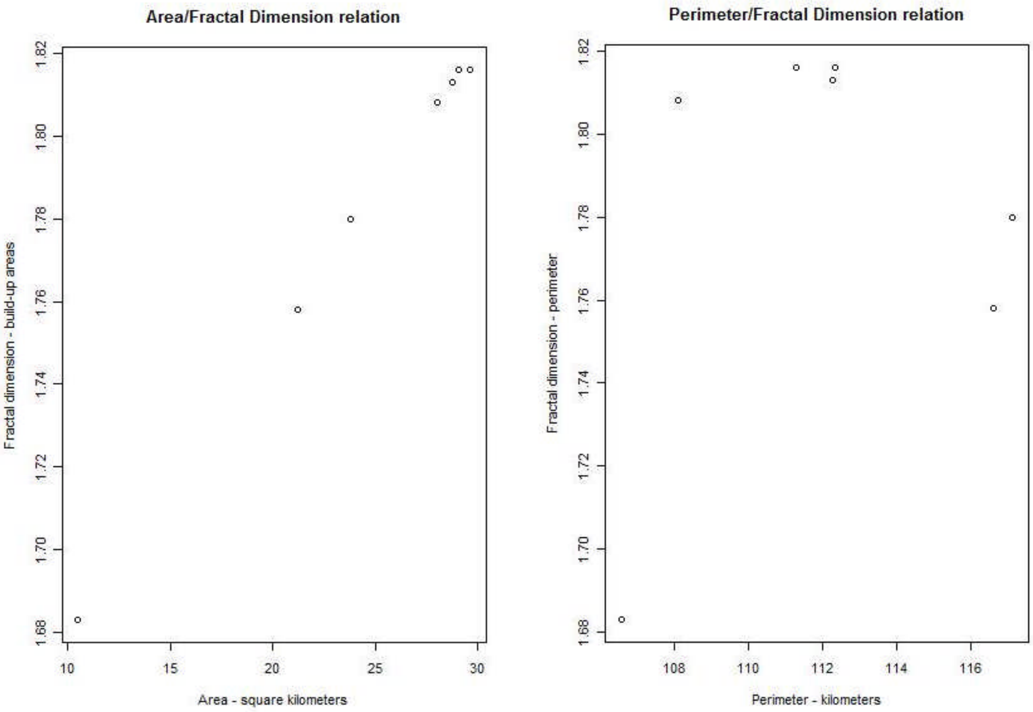

Fig. 3. Relation between area and its fractal dimension (left), and relation between perimeter and its fractal dimension (right).

The relation between area and its fractal dimension is strictly linear (Fig. 3 right). It enables to construct a linear regression model to describe precisely this relation. This model has a form:

*Fractal Dimension = 1.609103 + 0.007081 * Area*

As mentioned above, fractal dimension is a measure of complexity of shape. Although cities are more complex shapes with vertical dimension as well, their cartographical representation is planar and therefore is here examined as planar objects. And thus, plane has a fractal dimension equal to 2 and from the observed trend, it is obvious that growing area will result into fractal dimension equal to 2 – therefore the city will have covered the whole plane. By a simple calculation, the fractal dimension of 2 would correspond with area of 55.2 km^2.

Theoretically, this area value represents critical frontier for Olomouc development. Nowadays, the area is 29.6 km^2, so one can only guess what the future development will be. But there are two possible explanations (considering the hypothesis of linear relation between area and its fractal dimension):

- Predicted area 55.2 km^2 is a limit, which cannot be reached, the city will never grow up to this size,

- Once the city reached predicted area 55.2 km^2, it will continue to grow, but since its complexity in plane cannot grow any longer, the growth will be realized in vertical dimension

Important fact is that while the distance from the center of Olomouc to its peripheries is approximately 4 km, a circle with volume of 55.2 km^2 has radius of 4.2 km. Therefore a natural interpretation would be to expect Olomouc to grow only to 'fill' the non-build up areas within the distance of 4.2 kilometers from the city.

4. CASE STUDY 2 – FRACTAL ANALYSIS OF LAND COVER WITHIN OLOMOUC REGION

Visualization of land cover in Olomouc region, which has fractal structure typical for landscape, is shown in Fig. 4. Areas with maximal and minimal fractal dimension, both for artificial areas and natural areas, are also outlined. From artificial areas, the maximal fractal dimension (*D = 1.393*) has town Hlubočky (Mariánské Údolí) and the minimal value of fractal dimension has part of Bystrovany municipality (*D = 1.220*). In the first case, the maximal fractal dimension is caused by the topography of the town. Hlubočky (Mariánské Údolí) was built in steep valley on both sides of the river and thus is forced to follow highly irregular topography, which results into observed fractal dimension.

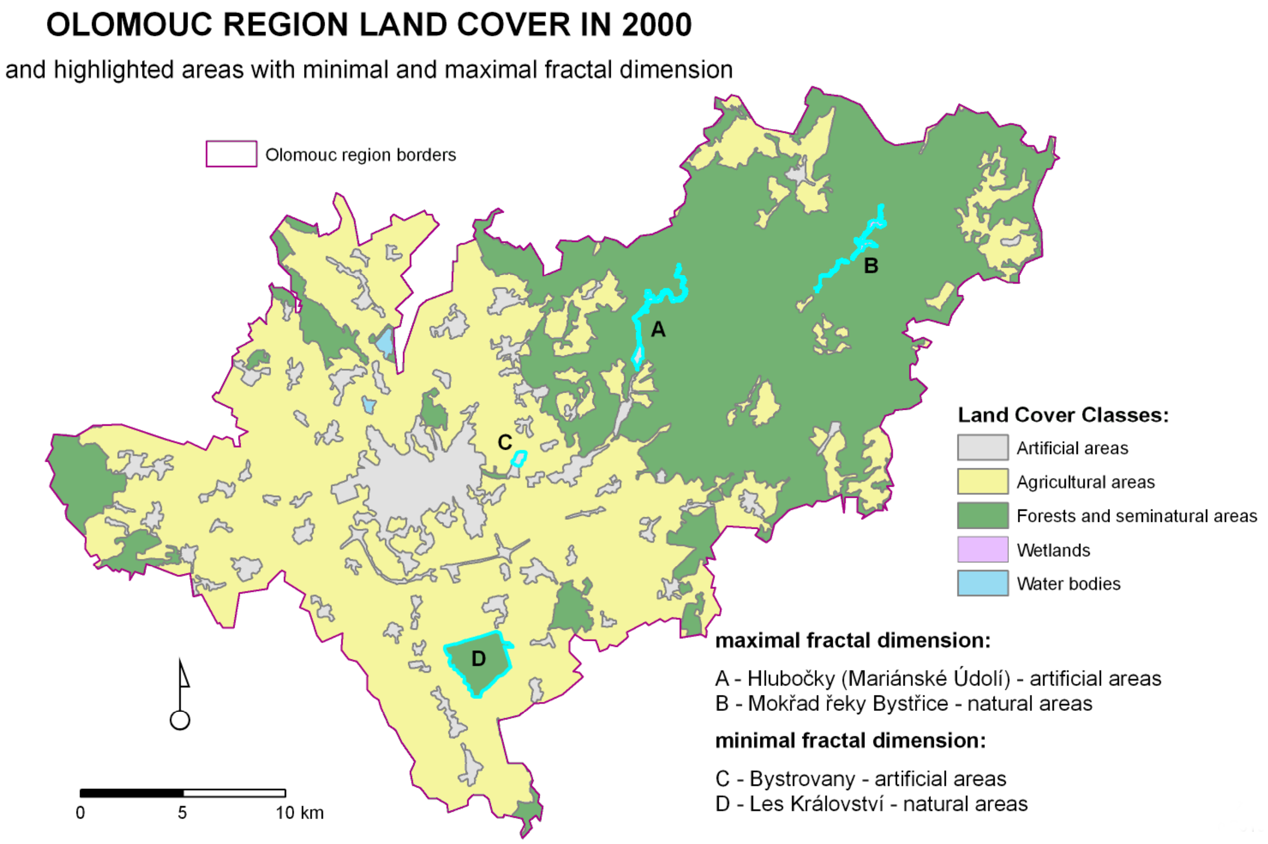

Fig. 4. Olomouc region land cover in 2000 and highlighted areas with minimal and maximal fractal dimension.

On the contrary, part of Bystrovany municipality represents distinct regular shape – almost square. There were no landscape borders or limitation when the settlement was built and regular fabrication of the build-up area (agricultural facility) was, probably, the most logical one. From natural areas, maximal fractal dimension has the wetland area of Bystřice river (*D = 1.396*), which is part of highlands with almost intact landscape. Very regular shape has forest southern from Olomouc called Les Království and its fractal dimension (*D = 1.193*) corresponds with that fact.

At last, join of all areas within class was accomplished and overall fractal dimension calculated. Results are shown in Tab. 2.

Tab. 2. Overall fractal dimension of particular land cover classes in Olomouc region.

Land cover class	Fractal dimension
Artificial areas	1,438574
Agricultural areas	1,385772
Forests and seminatural areas	1,350355
Wetlands	1,395799
Water bodies	1,263722

It is clear from Tab. 2 that highest fractal dimension have artificial areas, which represents in the very most cases man-made build-up areas (villages, towns, various facilities). Although knowledge how to plan and build up the settlement more properly was known long ago, urban sprawl emerged and has great influence on the irregular shape of artificial areas. Wetlands are very specific class, which are fully determined by natural processes and its fractal dimension is the highest among natural areas. On the other hand, water bodies have the lowest overall fractal dimension. It is necessary to note that line objects, which would fall into this class (rivers, streams, channels, etc.), are excluded due to CORINE classification methodology. And that is why the water bodies have this overall fractal dimension – only man-made or man-regulated water bodies were identified by the classification process and consequently analyzed.

To objectively prove the significant statistical difference among the land cover classes, the ANOVA was used. Before that, Shapiro-Wilk test was performed to check up the normality of data. It was confirmed and ANOVA could be used. It was then proven that mean fractal dimension values are significantly different and thus the classes are different too. One can then claim that classes (Tab. 2) originate from diverse processes. To acquire more detailed information, Student’s t-test was used to test significant difference only between two selected classes − Artificial areas and Forests and seminatural areas − to objectively prove previously anticipated fact that especially these two classes should be different. Test prove this significance and one can claim that by calculating the fractal dimension of land cover areas it is possible to study, evaluate and interpret the processes lying underneath the current land cover appearance.

According to the CORINE Land cover classification system and acquisition of the dataset in reference scale 1: 100,000, influence of generalization on the fractal analysis needs to be taken into account. The more generalized areas in land cover classes, the more regular their shapes. And the results of fractal analyses are less accurate (in sense of capturing objects as much realistically as it is possible). Furthermore, formula (3) implies that the longer perimeter of the shape, the higher fractal dimension as a result. And this is very important fact, when calculation using formula (3) is used. Fractal analysis results are then influenced by the factors from logic sequence:

reference scale of map/dataset – generalization degree – perimeter of an area – fractal dimension value

5. CONCLUSION AND DISCUSSION

Changes in urban form of the city of Olomouc by means of fractal dimension were observed. Detailed examination of relations between build-up area, perimeter and their fractal dimension, was made. A dependency between build-up area and its fractal dimension emerged, which may lead to interesting assumption about future development of the city. Consequently, the fractal dimension of the boundary of the city seems to concentrate around value 1.27, which is close value to the one of Koch curve, and thus city’s fractal physique is proven. Possible interpretations of the found dependencies were presented and brief discussion was stated. Fractal analysis of the city growth is now frequently used in urban planning and represents robust tool to enhance spatial analyses concerning urban topics.

Furthermore, the use of fractal geometry in evaluating land cover areas was presented. Resulting values of fractal dimension of such areas were commented using expert knowledge of the Olomouc region. Geographical context was mentioned too and proper visualization was made as well. Overall fractal dimension was calculated for comprehension amongst land cover classes. Finally, some important aspects of generalization influence and CORINE classification system on the results were mentioned.

The paper brings to the reader basics of fractal geometry and its possible usage in geospatial analyses. Brief historical facts are also presented and plenty of publications and papers noted. It is obligatory to introduce methodological frame of fractal geometry apparatus, including formulas by which the fractal dimension was calculated. Two original case studies were carried out to demonstrate practical use of fractal geometry and consequence analyses. As mentioned above, fractal analysis built its stable position in various natural sciences, including geoinformatics and geocomputation

Fractal analyses are very sufficient for measuring complexity or irregularity of various objects, but there are other metric characteristics of the shape (e.g. compactness, convexity, roundness, elongation and others) to evaluate objects, respectively areas in this case. But the main difference between fractal geometry and this group of metric characteristics is in use of mathematical apparatus and, what is even more important, in concept of fractal geometry and chaos theory. And that is why the fractal geometry built its position in all kind of geospatial analyses.

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