ANALYSIS OF THE GEOMETRIC PROPERTIES OF DERIVED MULTIRESOLUTION TIN TERRAIN MODELS WITH A VIEW TO MORPHOMETRIC ANALYSIS

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Abstract

Automatic creation of terrain models by simplifying detailed models become increasingly important with the availability of comprehensive (sometimes redundantly dense) elevation data. In many situations it is expedient to use a modified (simplified) model instead of an original model created from all available data. Simplification plays a key role in the generation of models in various resolutions. We derived TIN terrain models in a number of resolutions by selecting automatic model simplification procedures known from the field of computer graphics. The paper presents an analysis of ability these models for terrain modeling where geometrical accuracy is important – the accuracy of partial derivatives and from them derived morphometric parameters. Therefore, we focused on evaluating the accuracy of partial derivatives computed from the model. The main criterion is the deviation of the triangle normal to normal of a represented surface. We studied the impact of loss of detail in simplifying and impact of random error in input data. The selected simplifying procedures used detailed information about the shape of the terrain surface to adapt individual elements (triangles) to the represented part of the surface to achieve the best possible fidelity. It is shown that we can reach very high efficiency of the terrain model using surface simplification. The model with the fraction of elements (vertices, triangles) from the original one can maintain sufficient accuracy of derived geometric characteristics.

Keywords: simplification, terrain models, TIN, morphometric variables, comparison

1. INTRODUCTION

Modern methods of obtaining detailed data have brought the possibility to create efficient terrain models with high accuracy. This requires the inclusion of methods that have been previously used rarely, such as simplifying models. Large amount of data are processed to obtain detailed information about the shape of the surface, which will be used for the automated creation of the (derived) model, which efficiently and accurately represents the terrain surface. Simplification methods were developed and improved in other fields such as in the field of computer graphics. Nearly a decade ago, Luebke (2001) evaluated simplification methods in computer graphics as mature, enough to use in the field of terrain modeling.

Terrain modeling has its own specifics, which have to be taken into account in applying the procedures. Morphometric variables describe geometric characteristics of terrain and they are often entered into other application models. Therefore, an accuracy of morphometric variables is crucial. This reflects the accuracy of the values of partial derivatives and from them expressed morphometric variables. We will mention inaccuracies of partial derivatives determined from triangulated irregular network (TIN) models (polyhedral models based on 2D triangulation), which are essential for morphometric analysis. Just the used method of model accuracy assessment based on a normal deviation includes the characteristics of these inaccuracies. This part of the problem is treated in the second chapter of this paper.

The origin of numerical and positional deviations of calculating geometric characteristics of the TIN model presented Krcho (1999, 2001). The work (Feciskanin, 2009) contains extended analysis of the properties of deviations. There were also presented properties of the triangle properly configured to the represented surface, which ensures minimization of deviations of partial derivatives. Accordingly, we analyzed the most common methods and selected those that use the geometric characteristics of the properly configured triangle during the simplification process. We chose two methods under the assumption of a very good geometric fidelity: *quadric error metrics simplification* presented in the work by Garland and Heckbert (1997) and *memoryless simplification* presented in the work by Lindstrom and Turk (1998). The third chapter describes the basic characteristics of the methods and their relation to the definition of the correctly configured triangle.

We present an analysis of changes in geometrical fidelity measurements among the simplified models in distinct levels of detail in the fourth chapter. We also evaluated the impact of random error in input data. The results showed the positive impact of simplification to eliminate noise. They confirmed that the creation of a simplified model gives the opportunity to excel advantages of TIN, where it can very effectively and yet with sufficient accuracy represent a modeled surface on the basis of detailed information about it. There were obvious significant differences between the behaviors of the tested algorithms, despite their similar nature. This is evidence that the selection of an appropriate method has a major impact on the accuracy of the model.

The paper shows specific terrain analysis issues, specific TIN terrain models issues, and what is the value added to a similar analysis addressed by other research fields, for example (Surazhsky and Gotsman 2005). Analysis of algorithms, finding the simplification methods using geometrical properties of the optimal triangle and geometric fidelity analysis of created models by chosen methods are new contributions of this paper.

2. ACCURACY OF DERIVED GEOMETRIC PARAMETERS

The next part of analyses focuses on the creation of deviations in the calculation of the values of structural geometric parameters (slope, aspect, curvatures, and so forth) from a TIN model. Deviations present a result of the relationship between surface geometry and a shape of the triangle that it represents. The primary consequence of an inappropriate relationship is the triangle normal that does not match the terrain surface normal at a given location. Normal deviation cause inaccuracies in determining the partial derivatives and hence the calculation of morphometric variables.

2.1 Relationship between surface geometry and a shape of a triangle

This relationship can be studied on the basis of height differences Δz between the plane of the triangle σ_Δ and the terrain surface. For Δz is valid $\Delta z = P_z - P_\Delta$, where P_z is the height of the terrain at any point $P(P_x, P_y, P_z)$ and P_Δ is the third coordinate of the vertical projection of P on the triangle plane σ_Δ with coordinates (P_x, P_y, P_Δ).

An important role has curve $\Delta z = 0$, which is the intersection of the triangle plane and the terrain. Since the triangle vertices lie on the terrain and in the σ_Δ (a plane they determine), then also lie on the curve $\Delta z = 0$. This curve is a circum curve of the triangle. Its shape is subject to the shape of the terrain in a given location and it is very close to the shape of the Dupin indicatrix, which describes the shape of the terrain surface.

Another important indicator is the position of tangent point T of tangent plane σ_T that is parallel to the plane of triangle σ_Δ. Then, the triangle normal vector $\boldsymbol{n}_\Delta$ is identical to the normal vector of the tangent plane and then also to the normal vector of terrain $\boldsymbol{n}_T$ in point T. Tangent point T has the same values of the first partial derivatives, slope and aspect as calculated from the triangle. Feciskanin (2009) provides a more detailed characterization of the relationship between the surface geometry and a shape of the triangle.

2.2 Numerical and positional deviations

Krcho (1999, 2001) identifies a property of the triangle that is not suitable for the representation of terrain as an improper configuration of the triangle. There are deviations in calculations of the values of geometric parameters in improper configured triangles. Almost all triangles forming a triangular network incur less, or greater deviation. This is due to the fact that the real surface normal vector $\boldsymbol{n}_G$ in place of triangle centroid G – with horizontal position (G_x, G_y) – differs from the normal vector of triangle plane $\boldsymbol{n}_\Delta$ (see fig. 1). Therefore, it is valid $\boldsymbol{n}_G \neq \boldsymbol{n}_\Delta$ and the angle between normals is determined by the relation

$$\cos\delta n = \frac{\boldsymbol{n}_G \boldsymbol{n}_\Delta}{|\boldsymbol{n}_G||\boldsymbol{n}_\Delta|}. \qquad (\mathbf{1})$$

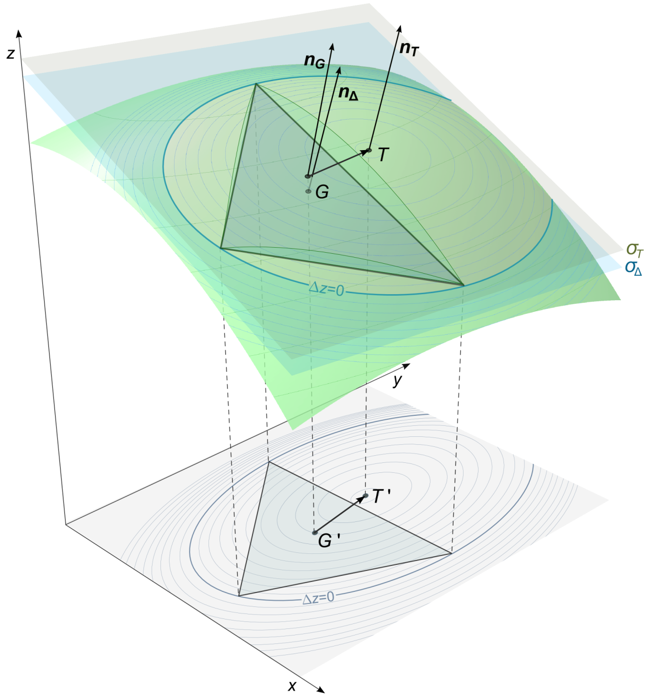

Fig. 1. Terrain surface geometry, triangle and deviations

Angle δn represents a normal deviation which, as mentioned above, causes inaccuracies in determining the partial derivatives and hence the calculation of morphometric variables. That is why we chose this parameter as the most important for evaluating geometrical fidelity with a view to morphometric analysis.

Deviations also have positional aspect. It can be usually found a point which has the same normal vector as a triangle plane, and therefore the same values calculated from the first partial derivatives. This point is the mentioned tangent point T of tangent plane σ_T that is parallel to the triangle plane σ_Δ. Positional difference between the point T and the triangle centroid G creates positional deviation vector.

2.3 Properly configured triangle

Due to the above principles and characteristics of numerical and positional deviations we can formulate the conditions for the proper configuration of triangle vertices to terrain surface so that the normal vector $\boldsymbol{n}_\Delta$ is identical with the real terrain surface normal vector $\boldsymbol{n}_G$ at the place of triangle centroid G. If $\boldsymbol{n}_G = \boldsymbol{n}_\Delta$ then there is no normal deviation and so $\delta n = 0$. This will ensure that values of the partial derivatives and from them expressed morphometric variables are identical as real values of terrain at G. This condition will be valid if the centroid G and the point T have the same horizontal location

$$G_x = T_x,\ G_y = T_y. \qquad (\mathbf{2})$$

Condition (2) is a theoretical condition for proper configuration of the triangle, formulated by Krcho (1999). This condition can be fulfilled only under certain conditions and only for the individual triangles, not for groups of triangles in a triangular network. This follows from the geometric relationship between the surface geometry and the shape of a triangle. We can effectively study the properties if the terrain surface, which could be described by the general function of two variables

$$z = f(x, y) \qquad (\mathbf{3})$$

is replaced in the small neighborhood of the triangle by a simpler surface. We used an osculating paraboloid with a tangent plane in its vertex identical to the tangent plane of terrain in selected point. The vertex part of the osculating paraboloid in differentially small neighborhood of point is identical to the terrain surface. We will consider small, but finite large neighborhood. Simplification of part of the terrain shape by substituting it with part of osculating paraboloid with vertex T can approximately, but sufficiently accurately reflects the studied relationship. The isolines of height differences Δz are then the intersections of osculating paraboloid and the plane parallel to the planes σ_T and σ_Δ, which are conics. In this case, the Dupin indicatrix gives exact characteristics of isolines of Δz.

For our needs will be more purposefully to define the conditions of the proper configuration for the plane triangle (local coordinates), not for scalar base (x, y). Thus, we will address all aspects of the conditions of the proper configuration in the plane. Then, the condition (2) shows that the triangle centroid G must lie in the center of a conic – intersection of a triangle plane σ_Δ and the osculating paraboloid. This can be achieved only if the conic is an ellipse. Intersection is then described by an ellipse with the center in the triangle centroid G, known as the circumscribed Steiner ellipse. In the places where Dupin indicatrix is a hyperbola the condition of proper configuration cannot be fulfilled without residue. The evidence can be found in (Feciskanin 2009).

We analyzed algorithms using the most common simplification methods and we selected those that use the presented geometric characteristics of the properly configured triangle during the simplification process. This creates a precondition for achieving a very good geometric fidelity of created models.

3. SIMPLIFICATION METHODS

There are plenty of works dealing with methods of simplifying models of surfaces and objects, dozens of them are described and categorized in survey (Heckbert and Garland 1997). Therefore, we only outline the major works that use the basic types of procedures.

3.1 Types of simplification methods

The majority of work presents a method to repeatedly perform a local change in geometry, while the limit reached chosen criterion. There are two basic approaches to the local changes leading to the creation of a simplified model. Methods can be divided into the categories, refinement and decimation. *Refinement methods* are starting with a few points from the original model in the initial state and more points (triangle vertices) are added while the accuracy of the model is not at the required level. On the opposite, *decimation methods* start with the original model and reduces the number of elements (vertices, edges, triangles), while the level of accuracy of the model according to the chosen criterion is sufficient. Advantage of decimation methods over refinement methods is the ability to compare the influence of geometry changes to the original model (not only the actual state of the model).

Some of the surface simplification methods have been developed specifically for terrain models. They use the fact that the terrain surface is considered under defined conditions as a function (3), where the height is only a function of position. This allows using 2D Delaunay triangulation and deviation measurement only in the direction of the axis z. Refinement methods are more common in this category. There are some traditional and still popular methods, namely, *Fowler and Little algorithm* (Fowler and Little 1979)*, VIP (Very Important Points) Algorithm* (Chen and Guevara 1987). From decimation methods is a well-known *Drop heuristic method* (Lee 1989). We can overall characterized traditional methods, which are parts of the GIS software solutions, as inefficient, often with insufficient accuracy. They are used mostly in creating a TIN model from a regular grid. For the purpose of creating a very detailed model of the elevation data is better to use other methods of simplifying models.

Simplification methods for more general models of surfaces and objects that cannot be described by the equation (3) are mainly decimation methods and refinement methods are used rarely. Refinement methods use mostly a hierarchical division of triangles, as it stems from a report (Heckbert and Garland 1997). Methods vary according to the approach to reduce the number of elements. Luebke (2001) categorized them into groups based on: vertex (or face) decimation, vertex clustering and vertex merging (edge contraction). Edge contraction (merge its two vertices V_0 and V_1 to one V) is most commonly used process. The advantage is that it is a more atomic operation than vertex decimation or clustering. In one step, one vertex, three edges and two triangles are removed (for non-border edges). It does not require the invocation of a triangulation algorithm.

3.2 Characteristics of selected methods

As mentioned above, we chose for evaluation *quadric error metrics simplification* presented in the work (Garland and Heckbert 1997) and *memoryless simplification* presented in the work (Lindstrom and Turk

1998). Both methods use an edge contraction to perform a local change in geometry. Two fundamental decisions affecting the properties of the final model. The first is the location of a new vertex after a contraction and the second is a way of sorting the edges – order of contraction based on selected measurement of edge weight. Despite differently defined conditions, both methods are very similar in the background.

The *memoryless simplification* method determines the weight of edges by calculating the change in volume when contracting edges. Weight value is the sum of volumes of tetrahedrons arising from the surrounding triangles shifting the original vertices V_0 and V_1 to the new vertex V (see fig. 2). The *quadric error metrics simplification* method determines the weight to the value of the quadratic distance of the vertex to triangle planes with vertices V_0 and V_1. Both methods need to first determine the location of the new vertex after edge contraction.

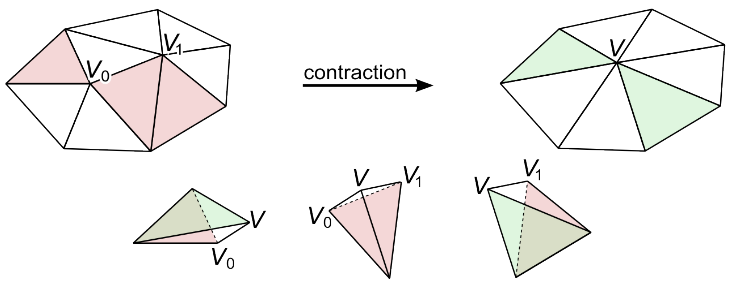

Fig. 2. Edge contraction in memoryless simplification

Determining the location of the vertex V by these methods is solved by the minimization of a function used to evaluate weight of edges. The process used in the *quadric error metrics simplification* leading in standard circumstances to unambiguously determine the location of V. This is the place with a minimum quadratic distance to triangle planes with vertices V_0 and V_1. These planes are so touching isosurface with a certain quadratic distance from the new vertex. This isosurface is quadric - ellipsoid, which gives the name of the quadric error metrics (Garland and Heckbert 1997). Example in two dimensions is shown in fig. 3.

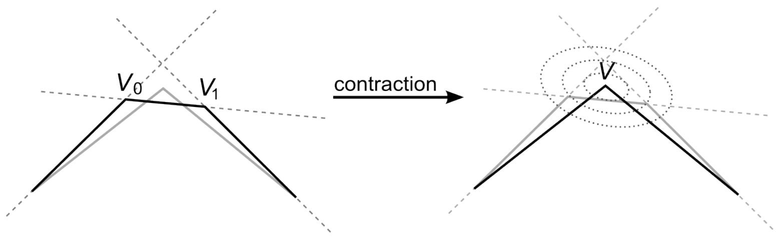

Fig. 3. Edge contraction in quadric error metrics simplification in 2D

The evaluation of tetrahedron volume change for determining the location of V is not enough. From a geometrical point of view it is the intersection of two planes. The *memoryless simplification* method adds a conservation of volume condition, which further defines a plane. In this case, the individual tetrahedrons have associated sign depending on whether it is an increase or decrease of the volume. This condition also ensures the global preservation of the model volume.

As pointed out by Lindstrom and Turk (1999), presented basic determination of the optimal position of the new vertex is very similar in both methods. Basically, they are using the same characteristics – can be converted into identical quadratic shape. It differs only in determining the weight of the triangles in a sum. While in the memoryless simplification method is the weight a square area of the triangle, in the quadric error metrics simplification method members are weighted equally, or by the triangle area.

An important difference, which concerns the behavior of the algorithm is that Garland and Heckbert (1997) calculated weight for each vertex is stored, while Lindstrom and Turk (1998, 1999) calculate weight on-the-fly. The most important factor that makes the quadric error metrics simplification method one of the fastest among similar methods is an easy determination of the weight of an edge contraction by sum of values stored per vertex.

3.3 Comparison with properties of properly configured triangle

Theory of approximation uses various evaluation methods and metrics for deviation measurement of surface approximation by triangles as plane triangular elements of the triangle network. On this basis, there were several works that try to express the optimal shape of a triangle regards the geometric characteristics of the represented surface. We will show that they are equivalent to those that we have defined in the second chapter and to triangles created by conditions in evaluated methods.

As stated by Melissaratos (1993), Nadler presented in 1985 that among all triangles with a given area a triangle optimal to replace the quadratic surface function is "long" in the direction of the minimum value of a second directional derivative and "narrow" in the direction of the maximum value of a second directional derivative. The optimum ratio ρ defined by Nadler is

$$\rho=\sqrt{\frac{\lambda_2}{\lambda_1}} \qquad (\mathbf{4})$$

where λ_1 and λ_2 are eigenvalues of Hessian of (3) (Heckbert and Garland 1999).

Heckbert and Garland (1999) define a triangle aspect ratio as the ratio of principal axes of an ellipse with the smallest area, which passes through the triangle vertices. The smallest area of any circumelipses of the triangle is Steiner ellipse (Weisstein 2005). Although not explicitly stated, the authors handle with parameters of Steiner circumellipse. They also showed that the aspect ratio that results from minimizing the quadric error metric

$$\rho=\sqrt{\frac{\kappa_2}{\kappa_1}} \qquad (\mathbf{5})$$

agrees with the optimum determined by Nadler. It is because eigenvalue λ_1 and λ_2 are the extreme values of normal curvature κ_1 and κ_2 and thus $\lambda_1 = \kappa_1$, $\lambda_2 = \kappa_2$ and equations (4) and (5) are equivalent. As mentioned before, memoryless simplification method minimizes metric with a great deal of similarity to the quadric error metric, so showed properties can be also applied to this method.

Extreme values of normal curvature κ_1 and κ_2 correspond to the principal axes of Dupin indicatrix. Their lengths are

$$r_1=\frac{1}{\sqrt{\kappa_1}} \text{ and } r_2=\frac{1}{\sqrt{\kappa_2}}. \qquad (\mathbf{6})$$

The above facts that it is required to have a triangle shape with Steiner circumellipse corresponding to Dupin indicatrix of modeled surface in the triangle centroid. This means that the above approach and so tested methods use the same characteristics as presented in chapter 2.3.

4. RESULTS

We analyzed geometrical fidelity of the simplified models created by *quadric error metrics simplification* and *memoryless simplification.* We compared deviations in distinct levels of detail and a particular degree of random error in input data.

4.1 Input

Artificial surface defined by mathematical function of two variables represented terrain surface. Its geometrical properties were similar to the geometrical properties of a common terrain surface. We used an artificial surface because of the possibility to calculate exact values of partial derivatives and morphometric variables. Then, calculated deviations are the real values of measurement without any other influence.

Input for simplification algorithms was a polyhedral model based on 2D Delaunay triangulation of a point set containing 68121 points. Each point was horizontally located in neighborhood of a regular grid node. The spatial distribution of input points was irregular, but they were distributed regularly over the surface. Each vertex had height calculated from chosen mathematical function. This point set contained points located directly on surface; there was no noise in their height values.

We created another two input point sets with random error in height value. Random error had normal (Gaussian) distribution with mean in real value. Two sets differed by size of random error determined by the standard deviation σ from the mean. For one set standard deviation was σ = 0.1 m, representing heights with a small amount of noise. The second point set has value of standard deviation σ = 0.5 m, representing heights with larger noise.

4.2 Simplified models

We generated simplified models in four levels of detail. Models were consisted of 5,000 vertices in the first level. Models had 1,000 vertices in the second level of detail, while third level was 250. The roughest models had 100 vertices. Althogh the algorithms could not guarantee the exact number of elements in the output, they were within 1-2 vertices of the target.

We used each method for generating model in every level of detail from all three input surfaces (without the noise, with small noise, with large noise). This means that we had 12 models generated by *quadric error metrics simplification* and 12 models generated by *memoryless simplification* for geometric fidelity analysis. Created models are presented in appendix (fig. 5).

4.3 Evaluation criteria and comparisons

We chose the normal deviation angle of a triangle δn as main criterion which was justified in section 2.2. We calculated arithmetic mean $\overline{\delta n}$ of δn for each model where triangles in distance below 50 meters from the border were not counted. This eliminated the border-effect (impact of inappropriate triangles along borders).

The structure of deviations helps detect their weighted mean $\langle\delta n\rangle$. Used weight was the area of triangles. The difference compared to the arithmetic mean suggests whether deviations are caused by small triangles (if $\langle\delta n\rangle < \overline{\delta n}$) or large triangles (if $\langle\delta n\rangle > \overline{\delta n}$).

Most common method for evaluating the geometric fidelity of the simplified model compared to the original is *metro* presented in the work (Cignoni, P., Rocchini, C. and Scopigno, R. 1998). It is based on Hausdorff distance. One of the characteristics given by *metro* method is the mean distance E_m between two surfaces. We used point sets of all input vertices (68121 points) of original model for calculating E_m.

We calculated these three parameters of geometrical fidelity for all simplified models. Values are presented in table 1 and fig. 4.

Table 1. Deviations of simplified models

		Memoryless simplification			**Quadric error metrics simplification**		
Vertices	$\boldsymbol{\sigma}$ [m]	$\overline{\delta \boldsymbol{n}}$ [°]	$\langle\delta \boldsymbol{n}\rangle$ [°]	$\boldsymbol{E_m}$ [m]	$\overline{\delta \boldsymbol{n}}$ [°]	$\langle\delta \boldsymbol{n}\rangle$ [°]	$\boldsymbol{E_m}$ [m]
100	0	3.27604	2.55572	1.79979	3.59204	2.88599	2.10333
	0.1	2.93885	2.57074	1.81072	3.28913	2.72489	2.05627
	0.5	3.33124	2.29989	1.70557	4.29413	4.02402	3.44086
250	0	2.34934	1.72327	0.63983	2.65791	2.04905	0.90578
	0.1	2.11402	1.72352	0.65308	2.45305	2.14613	1.00119
	0.5	2.31724	1.92601	0.68603	3.29024	3.13860	1.54991
1000	0	1.10905	0.89683	0.15340	1.35941	1.09395	0.22715
	0.1	1.18189	0.97909	0.16594	1.46036	1.35587	0.29514
	0.5	2.58018	1.77606	0.26611	1.88175	1.82220	0.43675
5000	0	0.53190	0.44372	0.03226	0.63173	0.53207	0.04846
	0.1	1.07265	0.83277	0.05657	0.90056	0.82604	0.06971
	0.5	6.22665	4.14721	0.22374	2.35582	2.03932	0.18641

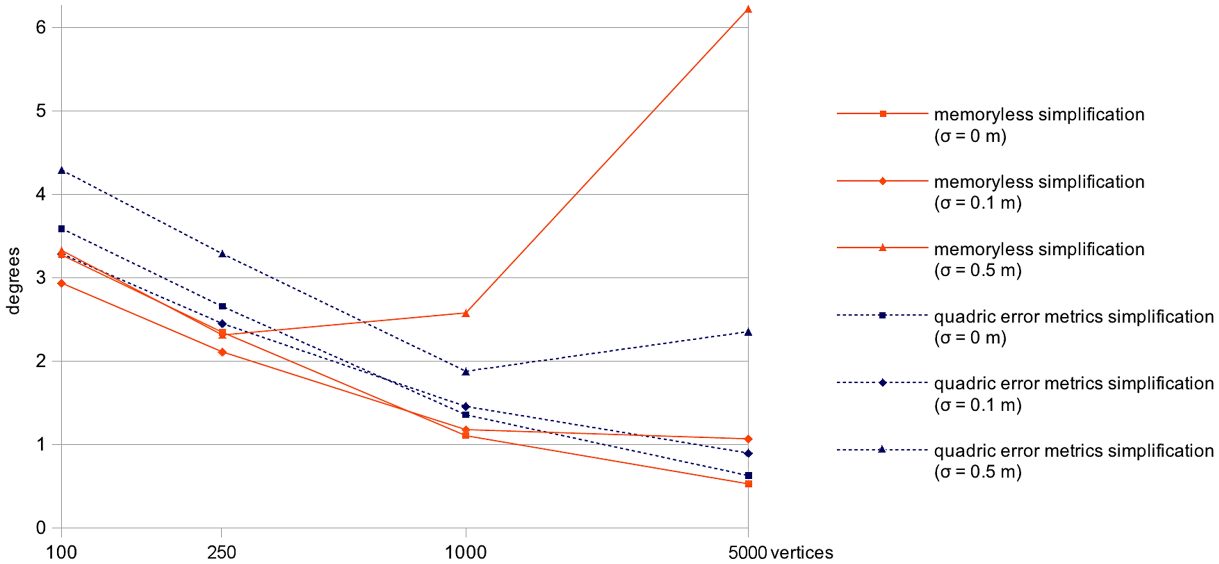

Fig. 4. Arithmetic mean of normal deviation angle of simplified models

As presented in the results, it is obvious that reducing the number of elements in the model decreases the geometric fidelity. This basic dependence affects the existence of random errors in input data. The random error of the heights affects derived geometric characteristics negatively at a higher density of points. That is why the same value of noise at the high number of vertices cause normal deviation increase. Input models made from 68121 vertices had mean normal deviations $\overline{\delta n} = 0.24184\,°$ for $\sigma = 0$ m, $\overline{\delta n} = 1.76868\,°$ for $\sigma = 0.1$ m and, $\overline{\delta n} = 8.64665\,°$ for $\sigma = 0.5$ m. From calculated errors it is apparent fundamental difference between the two methods to cope with noise in the input data. The *quadric error metrics simplification* method eliminates the uncertainty caused by random errors significantly better.

The analysis pointed to an interesting finding, such that in both methods, simpler models with low noise achieve the highest accuracy. We believe that this is due to the occurrence of narrow triangles created by simplification in the case of accurate information about the surface shape. This is demonstrated by comparing the arithmetic mean and weighted mean, where all models without random errors of heights had the biggest differences. Small values of random errors do not mean a major loss of information and force the algorithm to approximate the shape of the surface, which is then more smooth. This has a positive impact on the arrangement of triangular elements in the more simplified model.

The most important finding from the analysis of geometric fidelity is that the *memoryless simplification* method achieves higher geometric fidelity than the *quadric error metrics simplification* method. This is reflected by all three parameters used. As mentioned before, the only exception is the ability to deal with random errors in the input data with a higher number of elements.

5. CONCLUSION

Experimental results show the possibility to reduce the number of elements in the terrain model, while only to a relatively small extent reduces its geometric fidelity. We thus obtain a model with a significantly more effective operational capabilities than an original model. On the other hand, with well-used method of simplification will be given only the most relevant elements, allowing reveals the fundamental structure of the terrain. Moreover, it is possible to get different levels of detail and then highlight the hierarchy.

It was shown that simplification also significantly reduces the negative impact of random component of error in the input data. Under certain circumstances, even the simplified model increases the accuracy of derived values describing the geometric structure of the terrain. Results were impressive. Simplified model with approximately 1 % of vertices had the same value of $\overline{\delta n}$ as original model when $\sigma = 0.1$ m. Moreover, models consisting only of 100 vertices gives better results than the original model when $\sigma = 0.5$ m.

The selection of an appropriate method has a major impact on the accuracy of the model. Although the choice of two very similar methods processing the simplified model was different. Therefore, it is necessary to know the basic properties of algorithms for the proper selection of them. Based on the results, we recommend the *memoryless simplification* method. The same result found the authors Surazhsky and Gotsman (2005). By their comparison of a number of methods, for application where overall geometric fidelity is required, the most recommended is *memoryless simplification* method and the second choice is *quadric error metrics simplification*. It also confirms our assumption that the methods, which algorithms work with the geometric properties of the optimal triangle will achieve the best geometric accuracy.

New contributions of this paper include: Analysis and finding the simplification methods using properties of the optimal triangle; Analysis of simplifying capabilities of terrain model by selected methods at various levels of detail; Analysis of simplifying capabilities of terrain model by selected methods with various levels of random errors in input data.

In future work, we want to focus on analysis and improvement, or supplement to methods in those places where they show their weaknesses. Overall accuracy would increase the elimination of the most significant deviations rearranging critical triangles. An appropriate tool could be the edge flip optimization with specially selected optimization parameters. The aim is to move closer to creation of process for constructing the triangular network optimized for terrain analysis.

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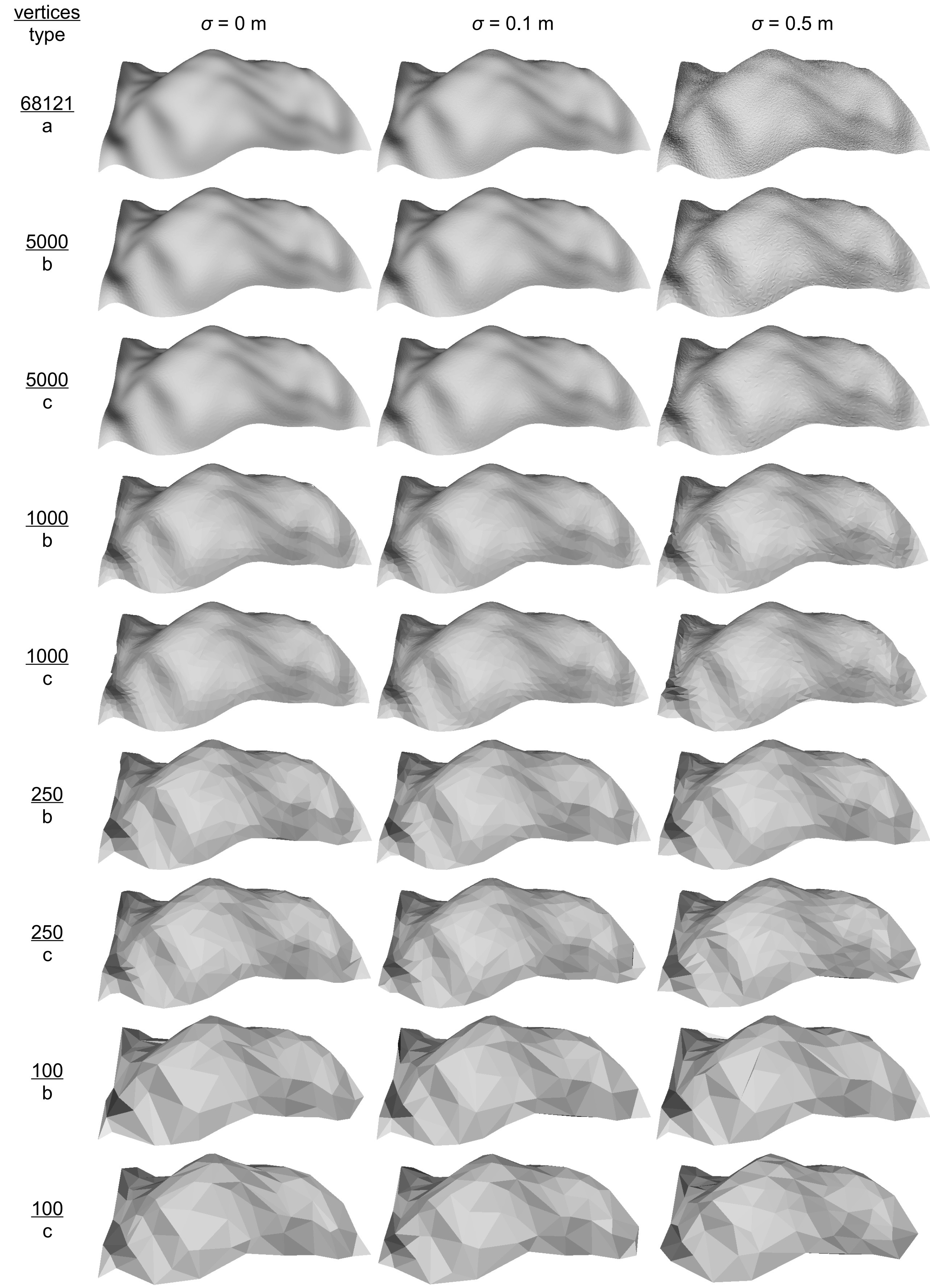
APPENDIX



a – original model, b – memoryless simplification model, c – quadric error metrics simplification model

Fig. 5. Compared models