MODELLING THE UNCERTAINTY OF SLOPE ESTIMATION FROM LIDAR-DERIVED DEM: A CASE STUDY FROM LARGE-SCALED AREA IN CZECH REPUBLIC

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ABSTRACT

This paper summarizes the approach and results of error propagation analyses in the Olše and Stonávka confluence area. In terrain analyses the outputs of the aforementioned analysis are always a function of input. Four different digital elevation model (DEM) resolutions (0.5, 1, 5 and 10 meters from LIDAR cloud points) were examined with Root Mean Square Error (RMSE) rating up to 0.317 meters (10 m DEM). They all originate in LIDAR survey. In the analyses was performed a stochastic Monte Carlo simulation with 100 iterations. Article targets the error propagation for large-scaled area using high quality input DEM. The DEM data uncertainty (RMSE) was computed from samplings and ground research (RTK GPS). According to empirical error distribution it was used semivariogram to model spatially autocorrelated uncertainty in elevation. Second procedure modelled the uncertainty without autocorrelation using random N(0,RMSE) error generator. Based on Monte Carlo simulation method the initial DEM was repeatedly perturbed by the uncertainty. Finally, statistical summaries were drawn to investigate the expected hypothesis. As expected; the error in slopes is increased with the vertical error in input DEM. According to similar studies using different DEM input data, high qualitative LIDAR input data decreases the output uncertainty. Errors without spatial autocorrelation do not result to greater variance in resulting slope error. Therefore it should be challenged, if error propagation without spatial autocorrelation represents sufficiently the true state of the nature of the error representation. In this case; although the slope error results (comparing random uncorrelated and empirical autocorrelated error fields) did not showed statistical significant difference, the input elevation error pattern has not been normally distributed and therefore the random error generator realization is not suitable interpretation of true state of elevation errors. The normal distribution was rejected because of the high kurtosis and extreme values (outliners).

Keywords: Uncertainty, Error propagation, Monte Carlo simulation, LIDAR-derived DEM, autocorrelation, RMSE, Slope estimation.

1. INTRODUCTION

Although many studies and research in field of digital elevation model uncertainty and its error propagation were done, still there are some unacceptable assumptions about the error expected. Firstly, the DEM error disappears with precise data acquisition and optimal interpolation algorithm. Secondly the DEM error is thought to be as small as not affecting the outputs of the analyses using DEM input. Last but not least DEMs are assumed and used as error-free models of reality, even though the existence of elevation uncertainty and gross errors are widely recognized [Oksanen, Sarjakosi 2005] [Torlegard et al. 1986]. In the last decades geomorphometry based on fine topscale DEMs have become popular in environmental science [Hutchinson, gallant 2000]. The accuracy of a digital elevation model is particularly important with its intended use [Erdogan 2010]. So the misjudgements increased the importance of solving DEMs uncertainty and the error propagation problem. The awareness that uncertainty propagates through spatial analyses and may produce poor results that lead to wrong decisions has triggered a lot of research on spatial accuracy assessment and data quality management in GIS (e.g. Heuvelink 1998, , Lee 1992, Shi et al. 2002) [Heuvelink 2007]. Information on the uncertainties in results from Geographic Information Systems (GIS) is needed for effective decision-making. Current GISs, however, do not provide this information. [HWANG 1998, Burrough 1993]. Furthermore there is the demand for presenting a level of accuracy (precision) [Burrough 1993]. Thus the long term vision in the research in spatial data uncertainty, accordingly DEM as well, has been to develop a general purpose "error button" for generating information systems (GIS) [Openshaw et al. 1991]. There are two main ideas how to implement this button. GIS could be incorporating the button into the product metadata [Goodchild 2000] or in more sophisticated solution is seen the button as user-dependent, which offers various possibilities for refining the error model according to the user's level of expertise [Heuvelink 2003]. The first steps towards the vision became a reality with building a data uncertainty engine, which implements the general framework for characterising uncertain environmental variables with probability models [Heuvelink, Brown 2005]. According to the authors many other research groups have worked on the design of an 'error-aware GIS', but very few have reached the operational stage. After the call for the development of geographical information systems that can handle uncertain data lasted at least for twenty years, Heuvelink developing the Data Uncertainty Engine (DUE) engine filled the gap [Heuvelink 2007]. Just the first step towards the solution of the error propagation problem has been made. The DUE must be further elaborated and improved. Sustained development of science and technology brought and will bring new methods of data collection and processing. As well DUE as other potential software, using different or same approaches, have to adjust to the changes. The usage of massive high-resolution DEMs based on airborne light detection and ranging (LIDAR) has renewed some assumptions. Two important factors appear to explain the lack of scientific knowledge about the use of LIDAR DEMs in uncertain-aware terrain analysis. Firstly, the common belief has been that the high quality of LIDAR DEMs [Hodgson et al. 2005, Barber, Shortridge 2005, Vaze, Teng 2007] will make the uncertainty-aware terrain analysis unnecessary. Secondly, uncertainty propagation studies have typically made use of simulation methods, such as simulated annealing and sequential Gaussian simulation [Goovaerts 1997], that are unsuitable for massive data sets because of their poor scalability [Oksanen, Sarjakoski 2010]. The aim of this paper is to analyze the aforementioned problems.

2. DEM ERROR

Spatial uncertainty is defined as the difference between the contents of a spatial database and the corresponding phenomena in the real world. Because all contents of spatial databases are representations of the real world, it is inevitable that differences will exist between them and the real phenomena that they purport to represent [Goodchild 2007]. Error is defined as the difference between reality and a representation of reality. In practice, errors are not exactly known. At best, there is known the distribution of values. The chances are equal that the error is positive or negative. [Heuvelink, Brown 2007]. The paper follows the taxonomy in which error is a measurable part of the uncertainty and is well-defined (probability density function is well known etc.) [Fisher 1999]. This choice is justifiable because the semantics of elevation do not suffer from the conceptual ambiguities common in, for example, defining the error in area class maps [Oksanen, 2005].The detailed process by which the errors in a DEM are created depends on the type of DEM and how it was created. Whatever method is used, DEM estimates are affected by several error sources, which can be grouped generally under three main classes: accuracy, density, and distribution of data, surface characteristics, and interpolation algorithms [Gong et al. 2000, Fisher 1998].Uncertainty in DEMs originates from two sources, errors in the lattice (gross, systematic, random) and accuracy loss due to lattice representation of the terrain [Li et al. 2005].It has been distinguish between positional and attribute uncertainty. Attribute uncertainty represents the deviation from true state of height and positional the shift in the object's position. Understanding the uncertainty is essential to correct modelling. Most frequently error in standard DEM products is reported as the Root Mean Squared Error (RMSE). Various methods have been used for estimating the RMSE. Most recently it is supposed to be estimated by comparison of elevations between the well located sites in a survey of higher accuracy with the elevation recorded in DEM at a minimum of 20 test points. The test points may be contour lines, bench marks, or spot elevations [Fisher 1992]. RMSE is based on the following formula:

$$RMSE = \sqrt{\frac{\sum (z-h)^2}{n}} \quad \textbf{(1)}$$

where *z* is the elevation recorded in the DEM; *h* is the elevation measured at the higher precision and *n* is the total number of tested locations (at least 20). The Gaussian error model (mean is the estimate of the true values and standard deviation is a measure of the uncertainty) makes only the most general assumptions about the processes by which the error has accumulated. [Hunter 1997]. To achieve an improved estimate of the error for any particular area, a set of measurements made at a higher precision is required, at best having another DEM of the same area at a higher precision. In this case it is possible to compare all values [Fisher 1998]. The spot heights and DEM or both DEMs have to be constructed separately, independence is strictly required. When additional information is available about the structure of errors in data set, the Gaussian model should be replaced with a substituting more accurate pattern of error (non-stationary or stationary spatial dependent random error field). According to previous studies (e.g. Lee 1992, Hunter 1997, Fisher 1998, Heuvelink 2003, Oksanen 2010, Caers 2011) DEM errors are spatially correlated, autocorrelation is a natural characteristic of the error data.
Hunter distinguished three cases of spatial dependence. Case one is spatial independence (r = 0). The elevation of each point is considered to be spatially independent of its neighbours (r = 0). In other words, knowledge of the error present at one point provides no information on the errors present at neighbouring points, even though the elevation themselves may have similar values. The elevation realization *h* at location *x, y* is achieved by disturbing each observed elevation *z* at same location by an independent disturbance term *N (0, RMSE)*, which is normally distributed random variable with mean 0and standard deviation RMSE (**Eq. 2**):

$$h_{(x,y)} = z_{(x,y)} + N(0, RMSE) \quad \textbf{(2)}$$

Case two is spatial dependence (limit r =1). At the other extreme, spatial autocorrelation reaches maximum. All errors are perfectly correlated, and there is in effect only 1 degree of freedom in disturbance field being applied to the DEM. It is unlikely, that any DEM production process would generate systematic error in elevations. Case three is the spatial dependence (0 < R < 1). The case of positive correlation less than 1 is clearly most realistic [Hunter 1997] and the disturbance *N(0,RMSE)* is spatially correlated to certain range following the fitted error model. Exponential [Holmes 2000] and Gaussian [Oksanen 2005] spatial autocorrelation models were selected to represent the correlation of the DEM error in the DEM uncertainty propagation studies. First exponential and later Gaussian model has been found to be realistic and suitable for topography [Goovaerts 1997]. The study investigates the type of model, range and the spatially independent random error pattern.

2.1 Error Propagation

There are two main approaches in error propagation of a continuous variable: the analytical and the numerical error propagation. The analytical error propagation method uses an explicit mathematical model to describe the mechanisms of error propagation for a particular multi-criteria decision rule [Eastman 1993]. In numerical methods, the calculations are not made with exact numbers. Instead of exact numbers are processed numerically generated random data sets. Usually they are generated on a computer and in a case of complicated data or physical model for analytical approach. Simulation of the error is made stochastically using Monte Carlo simulation, which is further subdivided into unconditioned and conditioned [Fisher, Tate 2006]. Unconditional error simulation models are based on number of realizations of random functions. At their most basic, they comprise an algorithm to select independent and uncorrelated values drawn from a normal distribution which can be added to the original DEM. The problem with unconditioned simulation is that it still makes the assumption that the pattern of error is uniform over the study area or a wider region. Conditioned error models directly honour observations of error at the sample locations. Such observations might have been obtained by comparison between the DEM and a higher accuracy reference data set collected from the same area [Fisher, Tate 2006]. In other words, the parameters of error model vary depending on the specific location. Comparing the results of using different methods of error modelling, the best method, which gives widely implementable and defensible results, is that based on conditional stochastic simulation [Fisher 1998]. The most common uncertainty propagation analysis approach makes use of Monte Carlo stochastic simulation [Johan Beekhuizen et al. 2009].The utilisation of Monte Carlo simulation, which is the most flexible method for investigation the propagation of uncertainty in terrain analysis, is time-consuming [Oksanen 2010]. Thus in this paper was used unconditional Monte Carlo simulation to propagate the error. Although the area is relatively small (11.26 km^2 respectively 1.25 km^2) and the relative difference in elevation less than 45 meters, it has been investigated the empirical error pattern to find anomaly or trend within (chapter 5.1).The outline of the Monte Carlo simulation is shown in the **Fig. 1**.

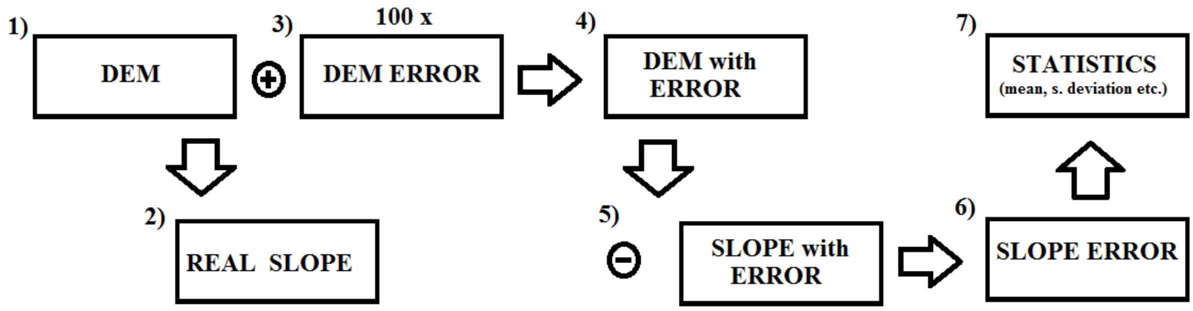

Fig. 1: Outline of Monte Carlo simulation, here 1) denotes the input DEM, 2) SLOPE calculated from 1),3) generated DEM ERROR,4) Alternative DEM, 5) Alternative slope, 6) Error in slope, 7) Statistics.

In simulations were used particular initial DEM. This DEM has been considered as error free representation of the true state of the elevation. Next has been calculated the “error free slope”. Then DEM error patterns have been generated according to initial DEM and error model attributes. Initial DEM has been perturbed with the generated random error field. The resulting DEM has the essential properties of both the error pattern and initial raster. Thus have been generated 100 realizations of DEM and subsequently slope estimates derived from alternative DEMs. Set of error patterns in slopes has been calculated as the difference between the error free slope and the particular alternative slope. Using appropriate statistics has been derived the results of the simulation. In some cases has to be used absolute error value instead of the error value.

2.2 Slope computation algorithm

A variety of methods can be used to estimate slope from DEM. Weighted least squares fit of a plane to a 3x3 neighbourhood centred on each point is the most amenable to a mathematical analysis of error propagation [Hunter 1997].Including the most used GIS software (SW) ArcGIS most of the GIS SW use this method to compute the slope from DEM. In this paper we decided to follow the aforementioned method’s algorithm. The output slope derivate can be calculated in degrees (angular unit **Eq. 8**) or percentage (**Eq. 7**). Degrees are the units chosen in the paper. Slope in degrees is calculated multiplying the slope in radians with 57.29578. Slope calculation (**Fig. 2**) is based on the change of height (rise) in the direction of x and y direction (run) - mathematically the first partial derivation of z in x and y axes. Thus the slope (**Eq. 5**) is determined by the rate of change (*Beta*) in both horizontal (*HD* **Eq. 3**) and vertical (*VD* **Eq. 4**) direction from the centre cell (*E*).

$$HD = \frac{\partial z}{\partial x} \quad \textbf{(3)}$$

$$VD = \frac{\partial z}{\partial y} \quad \textbf{(4)}$$

The approximation of the partial derivatives was made by third-order finite difference method (**Eq. 5** and **6**) [Skidmore 1989]. The method uses the 3x3 neighbourhood (**Fig. 3**) of the elevation values obtained in the raster around the centre cell. The distance between the elevation points is denoted *w*and represents also the cell (pixel) size of raster.

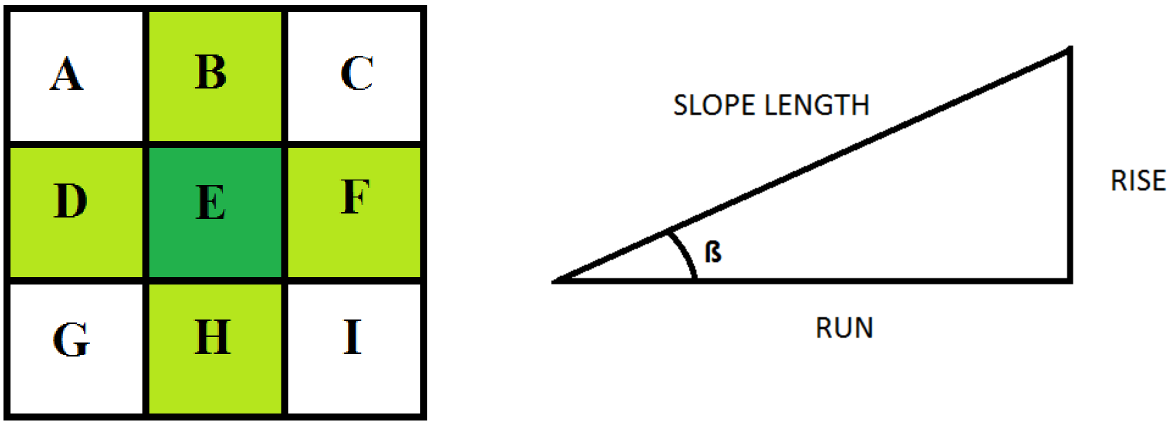

Fig. 2. Left the 3x3 neighbourhood window of the centre cell E and right the rise, run and beta description.

$$HD \approx \frac{(C + 2F + I) - (A + 2D + G)}{8 * w} \quad \textbf{(5)}$$

$$VD \approx \frac{(A + 2B + C) - (G + 2H + I)}{8 * w} \quad \textbf{(6)}$$

$$S = \sqrt{HD^2 + VD^2} \quad \textbf{(7)}$$

$$\beta = \arctan\left(\sqrt{HD^2 + VD^2}\right) \quad \textbf{(8)}$$

The influence of data precision on derived slope is highly related to the grid resolution. While using a high-resolution DEM (e.g. 1m grid resolution), the influence of data precision becomes quite significant. DEM resolution determines the level of details of the surface being described. It naturally influences the accuracy of derived surface parameters. On the other side usually the DEM error caused by data precision level is quite minimal, except in flat areas where the rounding errors could be significant [Zhou, Liu, 2004]. The precision significance has been investigated also, to prove or reject. We tried to minimize the rounding error, because of flat areas.

4 STUDY AREA

Error propagation was carried out along a 5.9 km stretch of the Olše River and a 3.2 km stretch of the Stonávka River. Both river sections are located in the northeast region of the Czech Republic near its border with Poland [Podhoranyi 2011].The area is located south of the city Karviná in the north-eastern part of the Moravian-Silesian Region. The area is 5.544 km in length and 2.281 km in width spaced. After the area affected with gross error has been eliminated, it remained a total area of 11.262 km^2. Because of gross errors and uncertainty in data collection process caused by atmosphere, three parts of the area (west) had to be clipped. Due the time-consuming computational method the 1.250 km^2 large study are have been used in case of higher precision data input (**Fig. 3**). The elevation of the area is varying between 211 and 256 (respectively 216 to 227 for small area) meters over the sea level. The slope is varying from 0°to 85° (respectively 0 to 67 degrees). The average slope values (1.95° to 3.9° respectively 3° to 3.5°) and the data histograms revealed the flat characteristics of the surface with few steep slopes

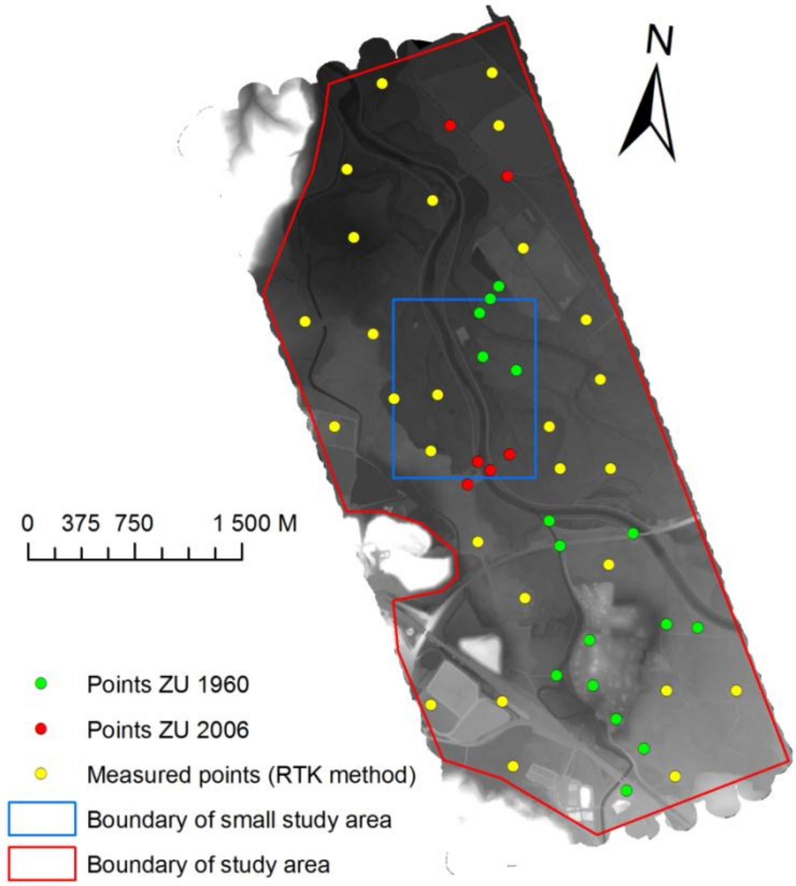

Fig. 3. Study area and measurement point locations for RMSE computation.

5. DATABASE CREATION

GIS database comes from various sources, each having its own level of uncertainty, depending on the specific technique used to acquire it. [HWANG 1998] The input data used to create the DEM in this study were obtained using the LIDAR method (Light Detection and Ranging). The Swedish company TopEyeAB, working with the MK-II laser system of its own design, carried out flights over the research area. The system consisted of a laser scanner with a50 kHz frequency, Inertial Navigation System (INS) and Global Positioning System (GPS) systems. The optical portion of the scanner deviated the laser beam into circular traces. The system was supplemented by a Rollei digital air camera with a 16-megapixel resolution (4080 x 4076 pixels). Scanning was carried out on the D-Hahn helicopter carrying the MKII-S/N 804 system at an altitude of 300 m [Podhoranyi, 2012]. DEMs (0.5, 1, 5 and 10 m. resolution) were computed independently of each other from particular acquired LIDAR data point cloud. The RMSE in input data has been calculated two times for every DEM to make comparison of possible inputs. First have been calculated the error values subtracting the DEM from the DEM with higher precision (resolution). 0.2 m resolution DEM has been used for 0.5 m resolution DEM. Then the RMSE (0.317 for 10m, 0.156 for 5m, 0.04 for 1m and 0.035 for 0.5 meter resolution) was calculated from the error values of the whole area. This RMSE values have been compared with the result of the second computation, which has been computed from RMSE of 49 point measurements in the study area (**Fig. 3**). 22 of 49 points were created by CUZK (Land Survey Office of Czech Republic) without any given information of data gathering method and accuracy. The second RMSE computation has a higher RMSE, which was effected by the location of the 49 points. They are not representative for the whole area. 49 points were located more in error prone surface (roadsides, river bank sides) as is their proportion of the total area. The 10 m resolution RMSE difference takes 5.7 cm (0.374 for 49 points and 0.317 for LIDAR), what is 17% of the total value of LIDAR RMSE. In other resolutions cases it was even worse (5 m – 14.1 cm, 1 an 0.5 m – 24.9 cm). It is necessary to mention, that the LIDAR DEM of higher accuracy inherent a certain uncertainty too. LIDAR RMSE results have been taken to fit the spatially uncorrelated error pattern as consequence of better representation of the continuous empirical error pattern. The autocorrelated error pattern has been made by investigating the empirical elevation error (*Chapter 5.1*).

5.1Simulation of random fields

The input error field has been made by investigating the empirical error pattern obtained with aforementioned method (*Chapter 2*). Error propagation has been modelled with and without spatially autocorrelated error field. The real state of nature is other than the expected theoretical state. First, there is an unjustified assumption that the mean error is zero [Li, 1988]. The error mean statistics were close to zero, but all of them were rejected as statistical zeroes using t-test hypothesis test in SW Statgraphics (**Tab. 1**).

Tab. 1 DEM error statistics (Number of Elevation Points, Error Mean [meters], Standard Deviation of Error [meters], Maximum Absolute Error [meters]).

DEM resolution	NUM. POINTS	MEAN	STD. DEVIATION	MAX ABS ERROR
10 x 10	263 520	$-3.2\ 10^{-2}$	0.692	11.942
5 x 5	1 051 997	$-1.2\ 10^{-3}$	0.362	12.053
1 x 1	26 289 516	$-2.3\ 10^{-3}$	0.085	9.567
0.5 x 0.5	83 963 724	$1.0\ 10^{-5}$	0.008	1.597

Thus the best fit is to follow the empirical model (Fisher 1998). If the difference between the elevation in the DEM and the actual surface (which equals the error surface) is done, the error surface should have a large positive autocorrelation [Goodchild 1995]. It is assumed that the RMSE over the study area is constant or spatially autocorrelated, what was confuted in previous researches (Fisher, Oksanen etc.). Although the total area is 11.262 km^2 small and according to the terrain surface and aforementioned research results (RMSE should be constant), it was necessary to divide it into smaller subareas, where this statement was proved. There have not been find a significant difference in parameters (range, partial sill and nugget). The area has been searched for trends. But none have been found. The best fitted model was the Stable one. According to previous researches it has been chosen Exponential and Gaussian to fit the pattern. Gaussian and Spherical had almost the same results, but the Gaussian better fitted the closest averaged values, so it has been chosen (Tab. 2, Fig. 4, 5, 6, 7). The appropriate shape of the spatial autocorrelation model was not critical as the computed autocorrelation parameters.

Tab. 2 Gaussian error model parameters.

DEM resolution	Lag Size [m]	Num. of Lags	Nugget [m]	Partial Sill [m]	Range [m]
10 x 10	6.480	12	0.246	0.165	49.025
5 x 5	5.054	12	0.068	0.042	31.090
1 x 1	3.610	12	0.001	0.005	13.752
0.5 x 0.5	0.544	12	$3.2\ 10^{-5}$	$2.7\ 10^{-5}$	3.897

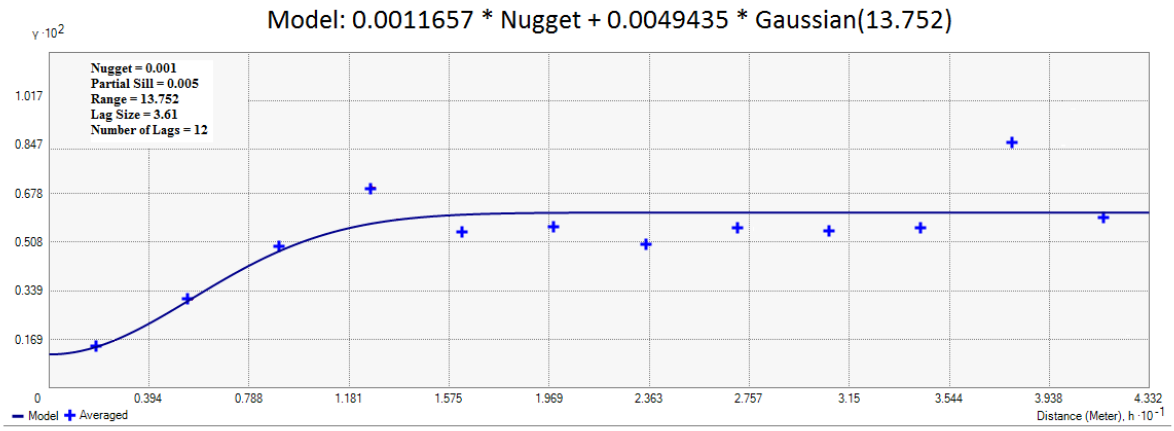

Fig. 4. Gaussian error model for 1x1 m resolution DEM.

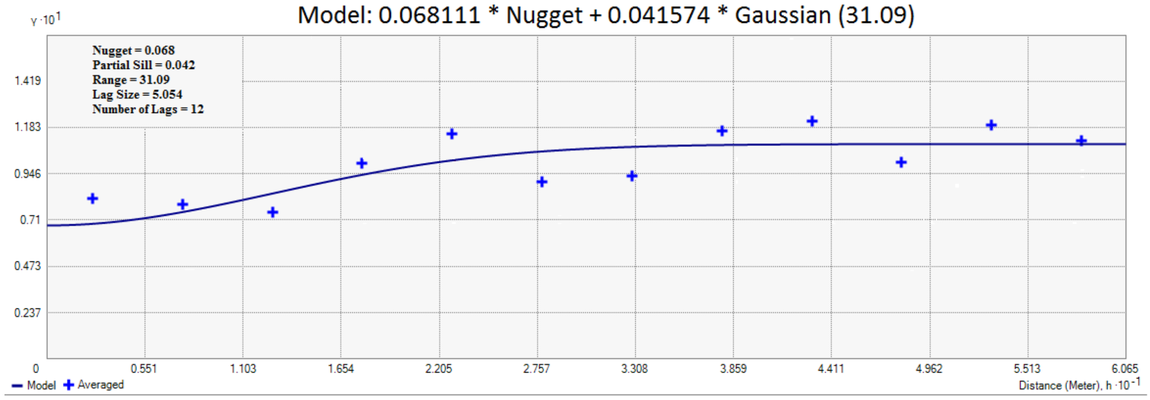

Fig. 5. Gaussian error model for 5x5 m resolution DEM.

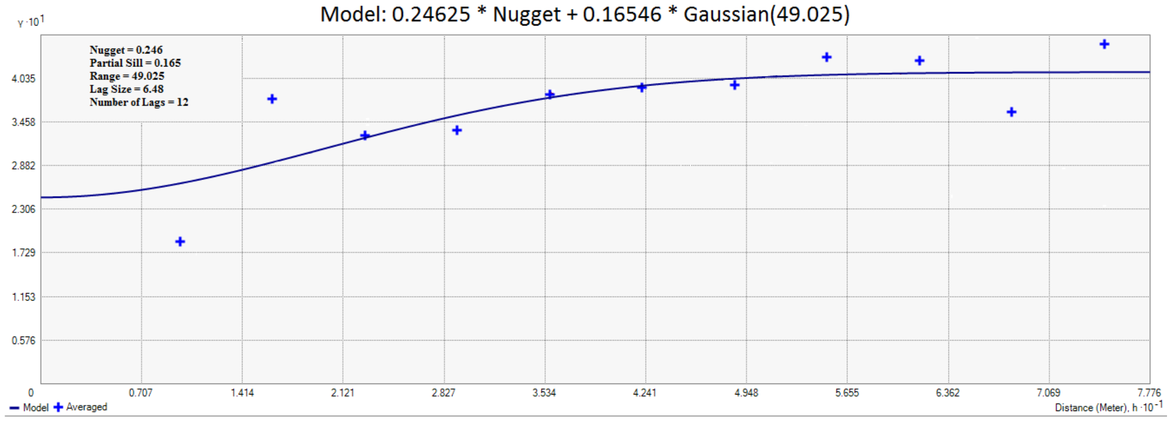

Fig. 6. Gaussian error model for 10x10 m resolution DEM.

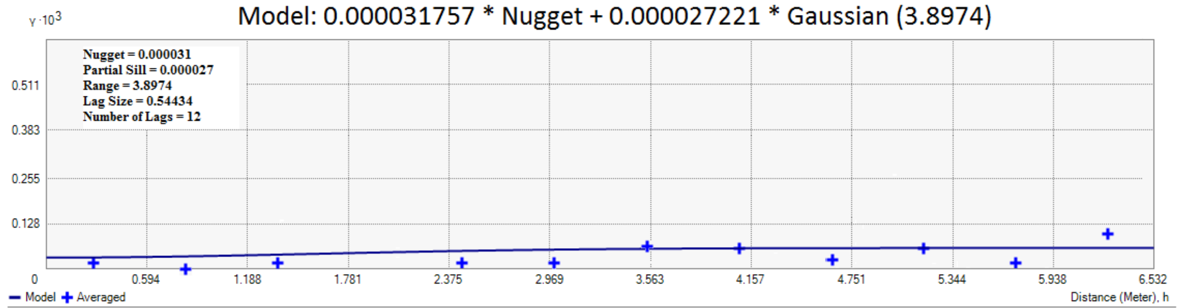

Fig. 7. Gaussian error model for 0.5x0.5 m resolution DEM.

The theoretical Gaussian models have been used to model the fields, **Fig. 8** depicts the difference between the spatially correlated and uncorrelated random field (10 m DEM). The error fields have been modelled 100 times for each DEM to perform Monte Carlo simulation. The outputs of aforementioned stochastic error propagation (**Fig. 1**) are mentioned in the following chapter results. The theoretical 0.5 m resolution Gaussian error model (**Fig. 7**) opens a question about the threshold, when is reasonable use spatially autocorrelated model and when just white noise.

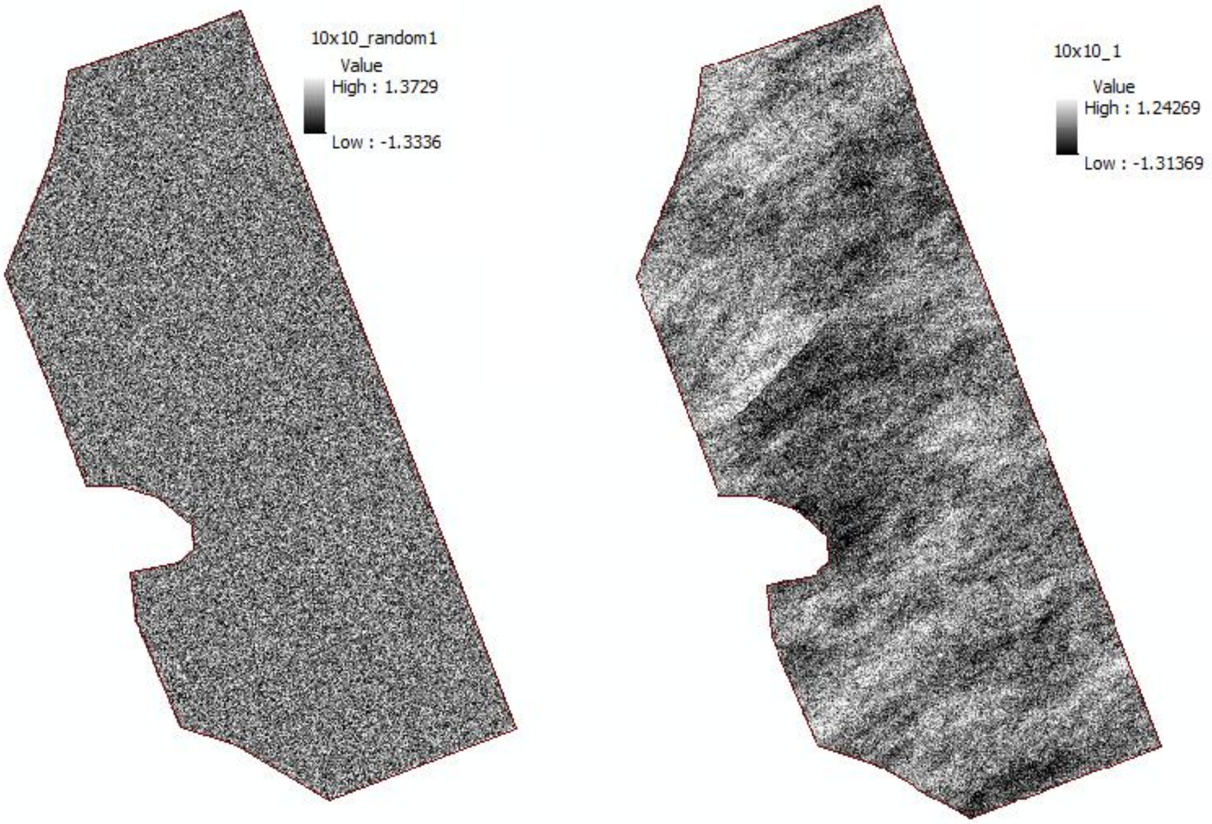

Fig. 8. Left uncorrelated white noise and right spatially correlated random error field of 10 m DEM.

6. RESULTS AND CONCLUSIONS

The error propagation results are summarized in table 3. For example in case of 10x10 m DEM error input we expect 0.680° (respectively 0.657° without spatial autocorrelation) error (mean of the means in column 5). For 5x5 m it is 0.657° (0.635°), 1x1 m 0.796° (0.776°) and for 0.5x0.5 m 0.272° (0.395°). The results are represented in absolute values. The behaviour of the error, that the value *x* and its opposite value *–x* represent the same deviation from the real state of nature, made possible this representation. It is more natural to see the errors in positive values and it enables better interpretations. If one number is most representative of the error evaluation, then it is the mean. It has been randomly selected 50 spot samples to prove the insignificant difference between the slope error result derived from inputs with and without autocorelation.

Tab. 3: Error propagation results [m or °] (Particular DEM, Input DEM error std. deviation for all elevation values, DEM RMSE, Output Slope absolute error statistics according to cells – mean, min, max, std. dev.).

DEM	**Auto-**	**Error in DEM**		**Output Slope absolute error [degrees]**			
resolution	**Correl**	**s. d.**	**RMSE**	**mean**	**Min**	**Max**	**std. dev.**
10 x 10	Yes	0.692	0.317	0.449-1.302	$2.98\ 10^{-8}$-0.392	0.979-4.312	0.373-0.881
10 x 10	No	0.692	0.317	0.379-1.296	0-0.273	1.169-4.925	0.283-0.829
5 x 5	Yes	0.362	0.156	0.296-1.255	0-0.366	0.580-4.473	0.11-0.882
5 x 5	No	0.362	0.156	0.216-1.286	0-0.313	0.631-4.543	0.167-0.831
1 x 1	Yes	0.085	0.04	0.276-3.017	0-0.469	0.805-13.205	0.202-2.276
1 x 1	No	0.085	0.04	0.286-1.441	0-0.361	0.919-5.747	0.210-0.871
0.5 x 0.5	Yes	0.008	0.035	0.043-0.510	0-0.134	0.126-2.438	0.029-0.385
0.5 x 0.5	No	0.008	0.035	0.027-2.587	0-0.164	0.120-10.651	0.073-1.557

Every spot sample has 100 alternative values, which have been used to compute mean and standard deviation. Two sample F test (st. deviation) and two sample t rest (mean) have been used. Null hypothesis set to: There is no difference in standard deviation (respectively means) and alternative hypothesis to: There is statistically significant difference between the std. deviations (means). For example for 1x1m resolution; we discovered that 49 in 50 cases for mean, respectively 47 in 50 for std. deviation are not significantly different (**Tab 4** showing 5 examples). Errors without spatial autocorrelation do not result to greater variance in resulting slope error (Oksanen got same results). Therefore it should be challenged, if error propagation without spatial autocorrelation represents sufficiently the true state of the nature of the error representation. In else we proved, that DEM error input without autocorrelation does not result (few exceptions) to greater error estimate of slope. Critical is the 0.5x0.5 resolution DEM error input, which leads to more inequalities. This phenomenon should be further investigated to understand the reason.

Tab. 4: Two sample F-test respectively t-test for 5 spots. Hypothesis (H_0) *(sigma1/sigma2 = 1.0)* concerning the ratio of the standard deviations of one spot sample of 100 observations for F-test, and hypothesis concerning the difference between the means (mean1-mean2 = 0.0, sigma1 and 2 input needed too) for t-test. (both 95.0% confidence level, P-value 0.05 and less rejects H_0).

Random Sample	**Slope**	**Autocor. Value**	**White noise Value**	**Hypothesis test:** F(t) statistics	P-v.	Null Hypothesis
1 std. deviation	17.536°	0.322	0.314	(F) 1.052	0.803	Do not reject, ratio = 1
2 std. deviation	11.232°	0.396	0.400	(F) 1.051	0.803	Do not reject, ratio = 1
3 std. deviation	5.950°	0.407	0.416	(F) 0.957	0.828	Do not reject, ratio = 1
4 std. deviation	0.137°	0.442	0.392	(F) 1.271	0.234	Do not reject, ratio = 1
5 std. deviation	0.226°	0.502	0.322	(F) 2.435	1.10^{-5}	Do reject, ratio <> 1
1 mean	17.536°	0.798	0.795	(t) 0.067	0.947	Do not reject, difference = 0
2 mean	11.232°	0.787	0.778	(t) 0.200	0.841	Do not reject, difference = 0
3 mean	5.950°	0.769	0.817	(t) -0.825	0.410	Do not reject, difference = 0
4 mean	0.137°	1.117	1.155	(t) -0.643	0.521	Do not reject, difference = 0
5 mean	0.226°	1.008	0.894	(t) 1.911	0.057	Do not reject, difference = 0

Although the result of input error without autocorrelation did not showed greater aberration, it is not suitable for elevation error pattern modelling. In fine topscale and microscale (Oksanen 2005) scale has the error pattern large positive autocorrelation. Furthermore in our case the outliers are responsible for rejection of Gaussian distribution. The outliners have to be also incorporated to the error model, what has not been done due to the lack of time. The average variance and mean of the errors in slopes is not strictly increasing with steepness of the slope (e.g. **Fig.9**). This causality should be further investigated; one of the reasons is the insufficient number of samples with steeper slope. The prevailing spatial distribution of slopes in study are is partially captured also in mean slope error (**Fig. 10**, **Fig. 11**). Input based on empirical elevation error (AC) describes better the error pattern and leads to more realistic and accurate spatial distribution of slope errors according to slope in study area. White noise (WN) input error field is closest to AC in minimum slope error distribution. Linear planar surfaces (roads etc.) are inadequately propagated. Planar surface is the most error prone type. According to similar studies (Fisher, Goodchild etc.) using different DEM input data, high quality LIDAR input data decreases the output uncertainty. In our case, the autocorrelated model fitted the error surface with exception of its outliers. There is a need to find a way how to include them. Extreme values are higher in case of theoretical model with autocorrelation; random number generator produces smaller extreme values also. Autocorrelation also expands the std. deviation of extreme values. On the one side the extreme elevation error values were found to be clustered around the steepest slopes, on the other side the steeper slopes has smaller slope error result with same elevation error input. Range of fitted empirical error model (49.6 for 10x10, 31.1 for 5x5, 13.8 for 1x1 and 3.9 for 0.5x0.5) was decreasing with higher resolution. We do assume that there should be a specific resolution limit value, where range is close to 0. Geostatistical modelling is very time consuming. We had to decrease the extent for the 0.5x0.5 and 1x1 meter resolution inputs. To compute one 1x1 meter DEM resolution error pattern (21 983 304 values in 5964 rows and 3686 columns) took 12 days and 17 hours (using 30 GB RAM and 4 processors Intel(R) Core(TM2)2 Quad CPU Q9300, 2.5 GHz). This computation requires super-computer.

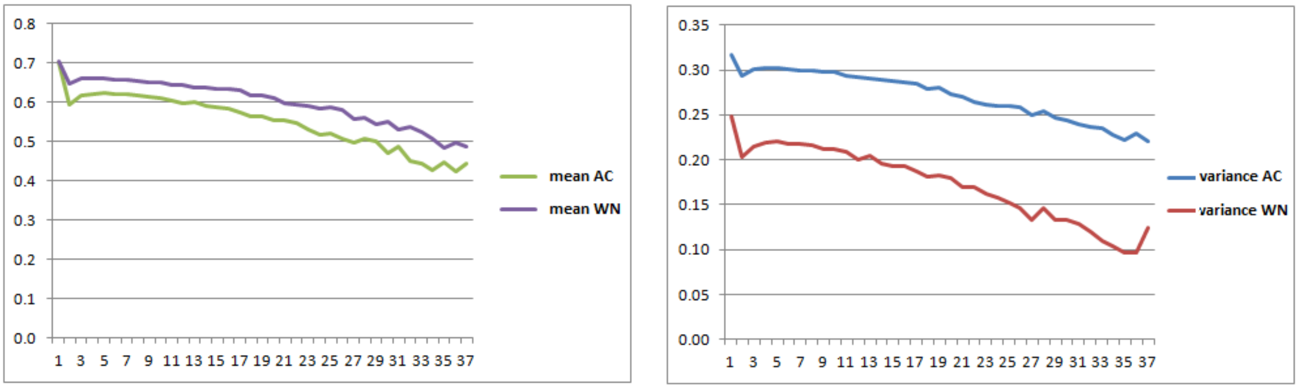

Fig. 9: Slope Error dependent variable (vertical axis) vs. Slope independent variable (horizontal axis) (WN randomly generated white noise, AC autocorrelation input according to empirical error pattern) showing the decrease in slope error with increasing slope.

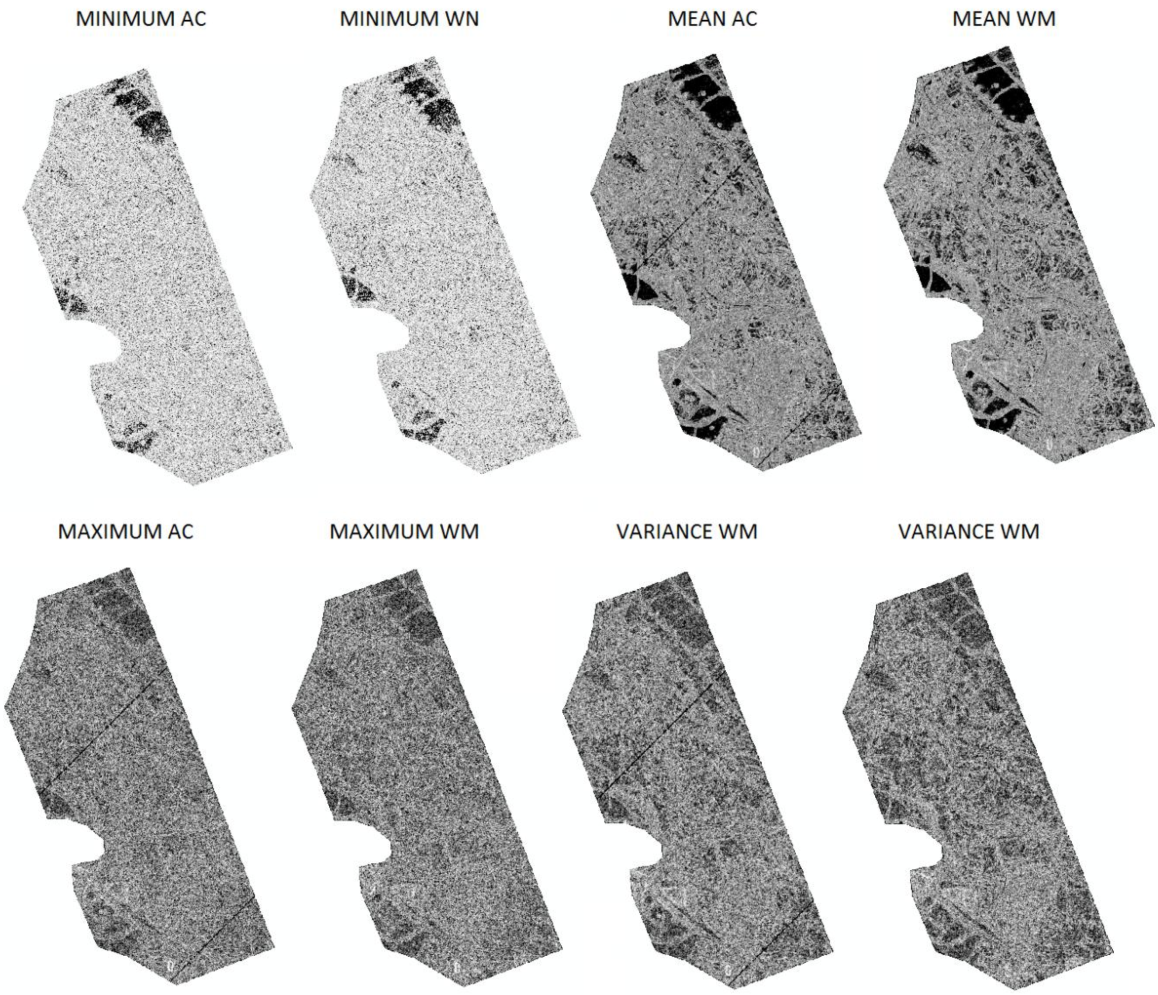

Fig.10: Mean, Variance, Minimum and Maximum statistics for 10x10m slope errors; if darker the colour then higher the slope error value and more planar the surface.

7. DISCUSSION

Although a lot of research has been made in the uncertainty and error propagation field over the last decades, still many questions left unanswered. In this study we focused to clear antagonistic results provided by Oksanen and Fisher. Oksanen declared that slope errors modelled without autocorrelation do not get worse result. In else, the slope derivate has not maximum variation with spatially uncorellated random error. On the other side Fisher declared, that the slope derivate computed from uncorrelated random error is worse, because of the poor input elevation error representation. We have found that Oksanen is right. Fisher is correct about the poor representation and the research area should be always investigated before analysed. We were not able to completely ascertain the character of the pattern error. Definitely it has been found the underlying error pattern. Some irregular outliners appeared which have to be incorporated. The next step should be investigation of the outliners. The empirical error model and the modelled error model have to be subtracted and the product investigated (external data may help too – underlying geology, terrain roughness, land use etc.). The resulting pattern is an addendum to the underlying error pattern. There can be more functions describing local shapes of error pattern. Sum of all functions (patterns) gives the resulting error pattern. We have found that there should be a threshold value, which in case of high precision and resolution data do not require the usage of autocorrelation in error surface (in case of high precision LIDAR data input and relatively small area).

It is true, that any given input data is carrying error value significant enough to change the resulting slope – even the high precision micro-scale LIDAR DEM. Results obtained with DEM inputs of same resolution and acquired with other methods (photogrammetric) could be used for better comparison and calculation of exact LIDAR improvement in slope error estimation. Other software tools should be used to prove simulated reality with gstat. According to time demanding computational process, it should be less consuming processes investigated for error pattern simulation, e.g. fuzzy approach. Software development and new supercomputers could be another solution. There is still a doubt, pros and cons, if unconditional Gaussian or sequential Gaussian simulation has to be used, how to model non-stationary error field in larger areas and what it is dependent on??

It is necessary to remember the main reason of dealing the uncertainty: decreasing the risk that the outcome will be incorrect and will lead to wrong decisions. This study has been made as error propagation background to inundation area delineation with GLUE method in the area. The processing of airborne hyperspectral data introduces uncertainty, which is enough to change the product. To know the uncertainty in the result is important in crisis management and other fields. Sometimes even one degree in slope can change the situation and flooded area.

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