EXAMPLES OF THE IMPLEMENTATION OF FUZZY MODELS IN TOURISM IN THE SOUTH MORAVIAN REGION

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Abstract

In geospatial information systems we often come across concepts expressing imprecision, incompleteness, uncertainty or vagueness, just like in everyday life. The degree of uncertainty or vagueness can be expressed through fuzzy set theory by membership functions. The fuzzy sets are more suitable for modelling of the vague phenomena than the classical crisp sets. We ordinarily find out spatial features which are not exactly bounded but are verbally determined. There are two examples of fuzzy exploitation in the South Moravian Region in this paper. Multicriteria decision making of tourism areas uses elementary fuzzy logic knowledge and assessment of bike trail difficulty which is considered according to the compositional rule of inference especially by Mamdani's method and defuzzification processes. The analyses apply the raster modelling using software ArcGIS 10.1, geoprocessing tools and programming language Python.

Keywords: GIS, fuzzy set, fuzzy logic, multicriteria decision making, fuzzy inference, modus ponens, compositional rule of inference, defuzzification, centroid, center of gravity, center of sums

INTRODUCTION

The term "fuzzy logic" was described by Lofti A. Zadeh in 1965. This many-valued logic characterizes wispy, unclear, vague, uncertain meaning [9]. In usual life we utilize unconfined terms such as steep slope, near the forest. We can speak about "linguistic variables" with linguistic values [10]. Real situations are modelled better by using fuzzy sets with uncertain boundary. Each element is in the set more or less. It is indicated by a degree of membership to a fuzzy set, by value between zero and one. Fuzzy sets are perceived as generalization of classical crisp sets which are their specific case. Quality "to be fuzzy" is often expressed as ambiguity, not as inaccuracy or uncertainty.

Definition of fuzzy set using the characteristic function.

Let $\boldsymbol{X}$ be a universe set (crisp set). A fuzzy set A of the universe $\boldsymbol{X}$ is defined by a characteristic function called membership function μ_A such that $\mu_A : \boldsymbol{X} \rightarrow \langle 0, 1 \rangle$ where $\mu_A(x)$ is the membership value of x in A.

The membership value assigns a degree of membership to a fuzzy set to any element.

$\mu_A(x) = 1$	element x belongs to a fuzzy set for sure
$\mu_A(x) = 0$	element x doesn´t belong to a fuzzy set for sure
$0 < \mu_A(x) < 1$	we aren´t sure if element x belongs to a fuzzy set.

Each function $\boldsymbol{X} \rightarrow \langle 0, 1 \rangle$ determines any fuzzy set definitely.

The membership degree to the fuzzy set is specified by mathematical functions [2].

We usually compose the membership functions of elementary linear functions. These are trapezoidal, triangular, S-shaped and L-shaped membership functions. We often use more complicated rounded functions, too as Gaussian function, bell-shaped function, sinusoidal function etc.

OPERATIONS ON FUZZY SETS AND FUZZY LOGIC

Operations complement, union and intersection on fuzzy sets are defined in similar way as on crisp sets [1].

The standard intersection of two fuzzy sets A and B is a fuzzy set with the membership function defined by
$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$. Zadeh´s intersection

The standard union of two fuzzy sets A and B is a fuzzy set with the membership function defined by
$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x))$ Zadeh´s union

The standard complement of fuzzy set A is a fuzzy set with the membership function defined by
$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ Zadeh´s complement

Functions for modelling fuzzy conjunction are called triangular norms (t-norms), for fuzzy disjunction triangular conorms (t-conorms). They are assumed as functions of two variables defined on a unit square.

Basic t-norms

$T_M(x, y) = \min(x, y)$ minimum t-norm

$T_P(x, y) = xy$ product t-norm

$T_L(x, y) = \max(0, x + y - 1)$ Łukasiewicz t-norm

$T_D(x, y) = \begin{cases} \min(x, y) & \text{if} \quad \max(x, y) = 1 \\ 0 & \text{else} \end{cases}$ drastic t-norm

The drastic t-norm is the smallest t-norm and the minimum t-norm is the largest t-norm, because we have $T_D(x, y) \le T_L(x, y) \le T_P(x, y) \le T_M(x, y)$.

Basic t-conorms

$S_M(x, y) = \max(x, y)$ maximum t-conorm

$S_P(x, y) = x + y - xy$ probabilistic t-conorm

$S_L(x, y) = \min(1, x + y)$ Łukasiewicz t-conorm

$S_D(x, y) = \begin{cases} \max(x, y) & \text{if} \quad \min(x, y) = 0 \\ 1 & \text{else} \end{cases}$ drastic t-conorm

The maximum t-conorm S_M is the smallest t- conorm, drastic t-conorm is the largest t-conorm, because we have $S_D(x, y) \ge S_L(x, y) \ge S_P(x, y) \ge S_M(x, y)$.

Now we can generalize expression of fuzzy sets union and intersection.

The intersection of fuzzy sets based on t-norm T is the fuzzy set with the membership function defined by

$\mu_{A\cap_T B}(x) = T(\mu_A(x), \mu_B(x))$.

The union of fuzzy sets based on t-conorm T is the fuzzy set with the membership function defined by

$\mu_{A\cup_S B}(x) = S(\mu_A(x), \mu_B(x))$.

Therefore, the standard intersection and union are special cases $A \cap B = A \cap_{T_M} B$ and $A \cup B = A \cup_{S_M} B$.

The fuzzy negation, the complement of the fuzzy set and various implications are defined similarly. [7].

Fuzzy relations

Let $\boldsymbol{X}, \boldsymbol{Y}$ be crisp sets. A binary fuzzy relation R from $\boldsymbol{X}$ to $\boldsymbol{Y}$ is any fuzzy subset R of the set $\boldsymbol{X} \times \boldsymbol{Y}$. Fuzzy relation R is described by the membership function $\mu_R : \boldsymbol{X} \times \boldsymbol{Y} \rightarrow \langle 0, 1 \rangle$.

We can define intersection on t-norm T and union on t-conorm S.

$$\mu_{A \cap_T B}(x, y) = T(\mu_A(x, y), \mu_B(x, y))$$

$$\mu_{A \cup_S B}(x, y) = S(\mu_A(x, y), \mu_B(x, y))$$

Definition composition of fuzzy relations

Let $\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$ be crisp sets, A, B binary fuzzy relations and T t-norm. Then sup-T composition of fuzzy relations A and B is fuzzy relation $C = A \circ_T B$ with the membership function $\mu_C(x, z) = \sup_{y \in Y} T(\mu_A(x, y), \mu_B(y, z))$.

FUZZY INFERENCE AND GENERALIZED MODUS PONENS

The fuzzy inference is a process which is applied to reasoning based on vague concept. The inductive method *modus tollens* and the deductive method *modus ponens* are the basic rules of inference in binary logic. In modus ponens we infer validity of a propositional formula q from validity of implication $p \Rightarrow q$ and validity of premise of a propositional formula p.

Generalized modus ponens

In fuzzy reasoning we use a generalized modus ponens according to following statement, where A, B, A', B' are fuzzy sets, X, Y linguistic variables. The scheme consists of a rule or a premise (prerequisite), an observing and a conclusion (consequence).

Rule if X is A, then Y is B
Observing X is A'
Conclusion Y is B'

The observing does not have to correspond to the premise in the rule. According to finding degree of comparison between premise X is A in the rule and current observing X is A' it happens modification conclusion Y is B in the rule and getting value B' of variable Y. If it is $A' = A$ in observing, it have to be valid $B' = B$. In fact, we operate more rules, input and output variables.

Example:

Rule if the slope is moderate, the bike trail difficulty is easy
Observing slope is steeper
Conclusion bike trail difficulty is harder

Compositional rule of inference

Practically we need to interpret verbal values of sets A, B mathematically and define the rule of fuzzy relation R between variables X, Y. We use the compositional rule of inference for assignment value B' of variable Y, which corresponds with value A' of variable X.

We can get term, where the set B' is the sup-min composition of the fuzzy set A' and the fuzzy relation R, written as $B' = A' \circ R$ with the membership [3]

$\mu_{B'}(y) = \sup_{x \in \boldsymbol{X}} \min(\mu_{A'}(x), \mu_R(x, y))$ standard intersection

or generally

$\mu_{B'}(y) = \sup_{x \in \boldsymbol{X}} T(\mu_{A'}(x), \mu_R(x, y))$ union based on t-norm T

Rule (X, Y) is $R(A, B)$

Observing X is A' compositional rule of inference on t-norm T

Conclusion Y is B', $B' = A' \circ_T R(A, B)$

We have to keep generalized modus ponens during relational reasoning, too, i.e. $A \circ_T R(A, B) = B$.

The fuzzy relations can be modelled by a logical implication or by a cartesian product T^* based on t-norm. We confine to the second possibility and we get:

$$\mu_{R(A,B)}(x, y) = T^*(\mu_A(x), \mu_B(y))$$

$$\mu_{B'}(y) = \sup_{x \in \boldsymbol{X}} \min(\mu_{A'}(x), T^*(\mu_A(x), \mu_B(y)))$$

We can generalize the properties to t-norm T :

$$\mu_{B'}(y) = \sup_{x \in \boldsymbol{X}} T(\mu_{A'}(x), T^*(\mu_A(x), \mu_B(y)))$$

If we choose $T = T^* = T_M$ we get:

$\mu_{B'}(y) = \sup_{x \in \boldsymbol{X}} \min(\mu_{A'}(x), \min(\mu_A(x), \mu_B(y)))$ *Mamdani's method*

For $T = T_M$ and $T^* = T_P$, it is: [5].

$\mu_{B'}(y) = \sup_{x \in \boldsymbol{X}} \min(\mu_{A'}(x), \mu_A(x) \cdot \mu_B(y))$ *Larsen's method*

MAMDANI'S METHOD

Let's have a look at Mamdani's method in detail [5].

Let $B = \{P_1, P_2, \ldots, P_k\}$ be a knowledge base with k rules for n input variables $X_1, X_2, \ldots, X_n$ and one output variable Y. Each of the variables X_i have the verbal value $A_{i,j}$ in j-th rule, variable Y has the verbal value B_j, where $i = 1,2, \ldots, n$, $j = 1,2, \ldots, k$. For Mamdani's regulator are defined:

Rules P_1 : if X_1 is A_{11} and X_2 is A_{21} and $\cdots$ and X_n is A_{n1}, then Y is B_1

P_2 : if X_1 is A_{12} and X_2 is A_{22} and $\cdots$ and X_n is A_{n2}, then Y is B_2

$\cdots$

P_k : if X_1 is A_{1k} and X_2 is A_{2k} and $\cdots$ and X_n is A_{nk}, then Y is B_k

Observing X_1 is A'_1 and X_2 is A'_2 and $\ldots$ and X_n is A'_n

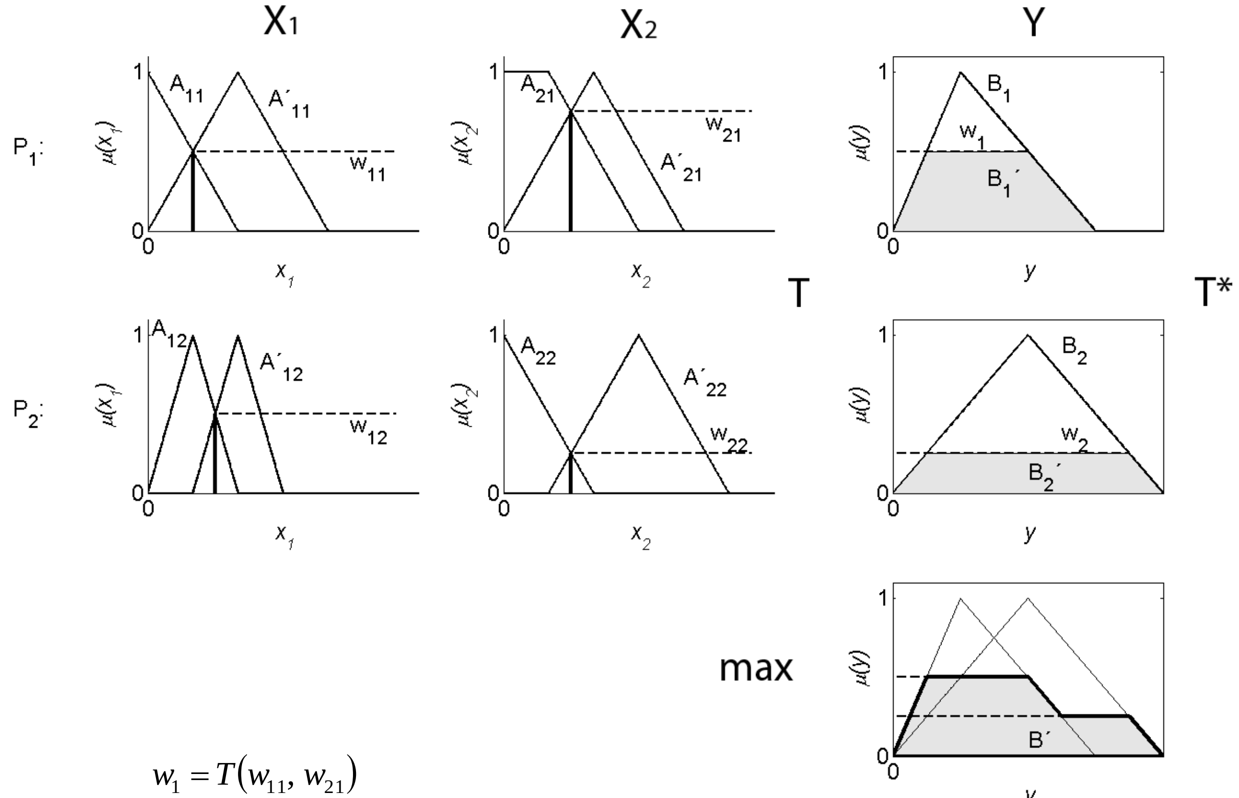
Conclusion Y is B'

Because the effort with the whole of the relation is numerically arduous, it is preferable to use the approach FITA (first inference then aggregation), which means reasoning of conclusion rule-by-rule, where the final aggregate conclusion is $B' = \bigcup_{j=1}^{k} B'_j$. Therefore $\mu_{B'}(y)$ can be presented as $\underline{\mu_{B'}(y) = \max_{j=1}^{k} \mu_{B'_j}(y) = \max_{j=1}^{k} \min(w_j, \mu_{B_j}(y))}$, where $w_j = \min(w_{1j}, w_{2j}, \dots, w_{nj})$ is the total weight of j-th rule, numbers $w_{1j}, w_{2j}, \dots, w_{nj}$ are particular degrees of fulfilment of the premises in j-th rule X_1 is A_{1j}, X_2 is $A_{2j}, \dots, X_n$ is A_{nj}.

We can generalize the properties to t-norm T.

Consider the generalisation of t-norm T for an intersection and t-norm T^* for an assignment of the relation (Fig. 1.). The membership function for degrees $w_j = T(w_{1j}, w_{2j}, \dots, w_{nj})$ is defined as $\underline{\mu_{B'}(y) = \max_{j=1}^{k} \mu_{B'_j}(y) = \max_{j=1}^{k} T^*(w_j, \mu_{B_j}(y))}$.

For Larsen's method is written $T = T_M$ and $T^* = T_P$.



$$w_1 = T(w_{11}, w_{21})$$
$$w_2 = T(w_{12}, w_{22})$$
$$\mu_{B'_1}(y) = T^*(w_1, \mu_{B_1}(y))$$
$$\mu_{B'_2}(y) = T^*(w_2, \mu_{B_2}(y))$$
$$\mu_{B'}(y) = \max(\mu_{B'_1}(y), \mu_{B'_2}(y))$$
$$\mu_{B'}(y) = \max_{j=1}^{2} T^*(T(w_{1j}, w_{2j}), \mu_{B_j}(y))$$

Fig. 1. Illustrative scheme of the universal regulator with two rules, two input variables and one output variable

DEFUZZIFICATION

If we apply crisp inputs, the results of inference are fuzzy outputs. We often need to find the particular real value of output by defuzzification. There are several methods to defuzzify. We can distribute them to methods searching the most acceptable solution and methods of the best compromise [6].

The methods of the most acceptable solution are presented by the methods of the most important maximum with selection of the biggest value of the membership functions placed leftmost, middlemost or rightmost - Left of Maximum (LoM), Mean of Maximum (MoM), Right of Maximum (RoM).

Methods of the best compromise include:
Center of Gravity (CoG) – the centroid of area (the centroid of the plane figure given by union of the part areas bounded by particular membership functions).
Center of Sums (CoS) – the centroid of sums (the centroid of the plane figure given by function, which is equal to the sum of the particular membership functions in the rules)
Center of Maximum (CoM) – the centroid of singletons (the centroid of the typical values, e.g. MoM, for the particular membership functions of the rules).

CoG

It makes for finding the first coordinate of the centroid of area bounded by the membership function $\mu_{B'}$. The method is mathematically difficult because we need to know the membership function and calculate the Riemann integrals. In the reasoning of conclusion rule-by-rule $B' = \bigcup_{j=1}^{k} B'_j$ is $\mu_{B'}(y) = \max_{1 \le j \le k} \mu_{B'_j}(y)$. The situation is simpler, if the universe of the output variable is discrete subset of real numbers $Y = \{y_1, y_2, \dots, y_r\}$.

$$y_{B'}^{CoG} = \frac{\int_{\boldsymbol{Y}} \mu_{B'}(y)\, y\, dy}{\int_{\boldsymbol{Y}} \mu_{B'}(y)\, dy} = \frac{\int_{\boldsymbol{Y}} \left(\max_{1 \le j \le k} \mu_{B'_j}(y) \right) y\, dy}{\int_{\boldsymbol{Y}} \left(\max_{1 \le j \le k} \mu_{B'_j}(y) \right) dy}$$ continuous membership function

$$y_{B'}^{CoG} = \frac{\sum_{i=1}^{r} \mu_{B'}(y_i) y_i}{\sum_{i=1}^{r} \mu_{B'}(y_i)}$$ discrete membership function

CoS [3]

It serves to find the first coordinate of the centroid of area which is bounded by the function defined as sum of the membership functions $\mu_{B'_j}$. The method is easy-to-use because it does not need to determine the conclusion B'. If the particular conclusions of rules do not overlap, the result of the method CoS is the same as for the method CoG.

$$y_{B_j'}^{CoS} = \frac{\int_{\boldsymbol{Y}} \left(\sum_{1 \le j \le k} \mu_{B_j'}(y) \right) y\, dy}{\int_{\boldsymbol{Y}} \left(\sum_{1 \le j \le k} \mu_{B_j'}(y) \right) dy} = \frac{\sum_{1 \le j \le k} \left(\int_{\boldsymbol{Y}} \mu_{B_j'}(y)\, y\, dy \right)}{\sum_{1 \le j \le k} \left(\int_{\boldsymbol{Y}} \mu_{B_j'}(y)\, dy \right)}$$ continuous membership function

$$y_{B_j'}^{CoS} = \frac{\sum_{i=1}^{r} y_i \sum_{j=1}^{k} \mu_{B_j'}(y_i)}{\sum_{i=1}^{r}\sum_{j=1}^{k} \mu_{B_j'}(y_i)}$$

discrete membership function

CoM

The first coordinate of the membership function is written for each conclusion of rule by the method of the most important maximum (Mean of Maximum) and the result is the centroid of singletons.

$$y_{B_j'}^{CoM} = \frac{\sum_{j=1}^{k} y_j \cdot \mu_{B_j'}(y_j)}{\sum_{j=1}^{k} \mu_{B_j'}(y_j)}$$

MULTICRITERIA DECISION MAKING OF TOURIST AREAS

The multicriteria decision analysis was required by officials of The Department of Regional Development to make out the most important tourist areas especially for family with children in the South Moravian Region. This assignment was determined by several required parameters. In our aspect we wanted to consider healthy environment, forest accessibility, proximity of bodies of water, optimum distance of the important roads, near historical and cultural monuments and density of the bike and hiking trails.

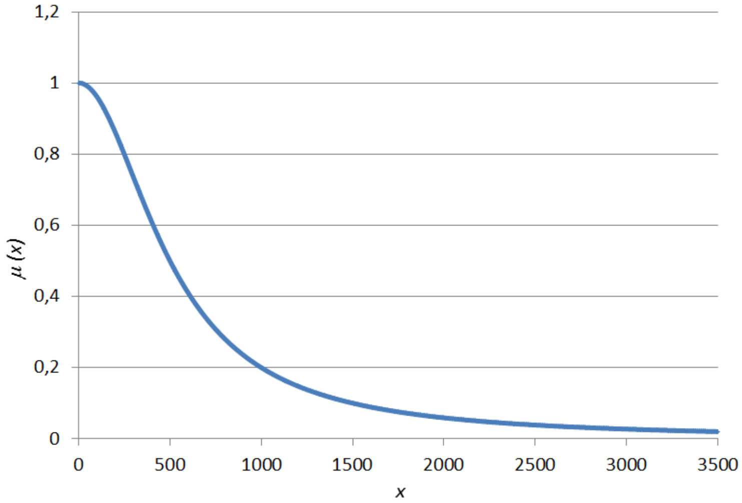
The evaluation was made both by fuzzy sets and crisp sets in order to compare them. The fuzzy raster maps were the results. The choice of the expression of the membership functions was performed by software to plotting mathematical curves. Below there are mentioned particular criteria, data sources (RA – regional authority data), qualities and units. Then the selected tools from Spatial Analyst Tools with their configuration follow.

Environmentally significant areas (Ea) – protected areas, natural parks (data RA, distance, meter)

Overlay – Fuzzy Membership – Small (midpoint 500, spread 2)

$$\mu(x) = \frac{1}{1+\left(\frac{x}{500}\right)^2}$$

(Fig. 2.)



for crisp sets

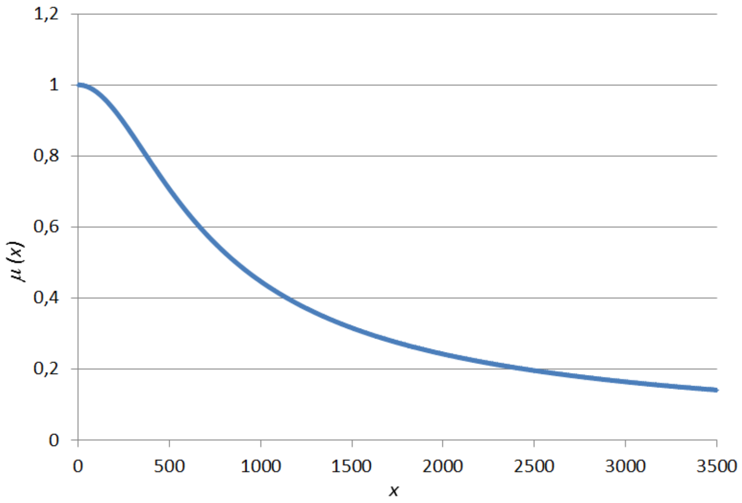
$$\mu(x) = \begin{cases} 1 & \text{if} \quad x \le 500 \\ 0 & \text{if} \quad x > 500 \end{cases}$$

Fig. 2. Membership function - environment

Forests accessibility (Fa) (data CEDA StreetNet, distance, meter)

Overlay – Fuzzy Membership – Small (midpoint 500, spread 2) – Somewhat (dilatation)

$$\mu(x) = \sqrt{\frac{1}{1+\left(\frac{x}{500}\right)^2}} \qquad \text{(Fig. 3.)}$$



for crisp sets

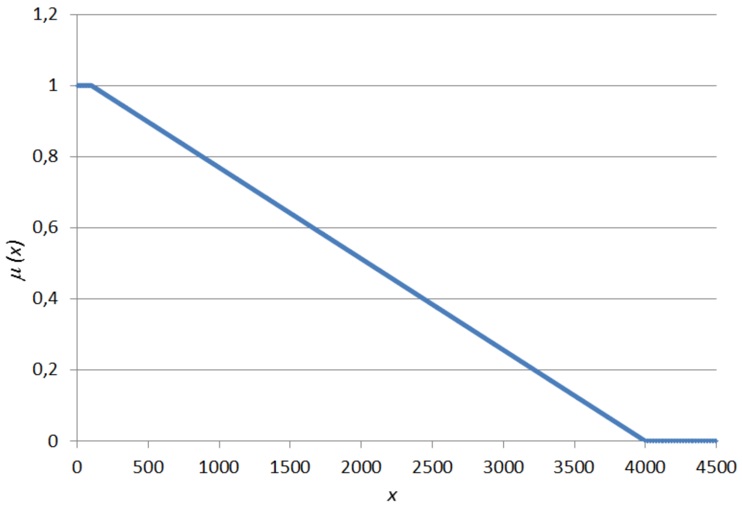
$$\mu(x) = \begin{cases} 1 & \text{if} \quad x \le 500 \\ 0 & \text{if} \quad x > 500 \end{cases}$$

Fig. 3. Membership function - forests

Bodies of water (Bv) (data CEDA StreetNet, distance, meter)

Map Algebra – Raster calculator

$$\mu(x) = \begin{cases} 1 & \text{if} \quad x < 100 \\ \frac{4000-x}{3900} & \text{if} \quad 100 \le x \le 4000 \\ 0 & \text{if} \quad x > 1000 \end{cases} \qquad \text{(Fig. 4.)}$$



for crisp sets

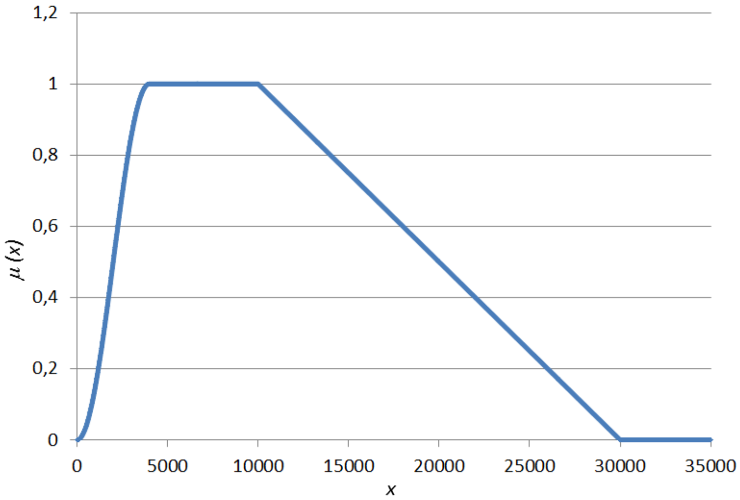
$$\mu(x) = \begin{cases} 1 & \text{if} \quad x \le 2000 \\ 0 & \text{if} \quad x > 2000 \end{cases}$$

Fig. 4. Membership function – water

Roads (Ro) – motorways and national roads (data CEDA StreetNet, distance, meter)

Map Algebra – Raster calculator

$$\mu(x) = \begin{cases} x \leq 4000 & \text{if} \quad \frac{1}{2}\left(1-\cos\left(\pi \frac{x}{4000}\right)\right) \\ 4000 < x < 10000 & \text{if} \quad 1 \\ 10000 \leq x \leq 30000 & \text{if} \quad \frac{30000-x}{20000} \\ x > 30000 & \text{if} \quad 0 \end{cases} \qquad \text{(Fig. 5.)}$$



for crisp sets

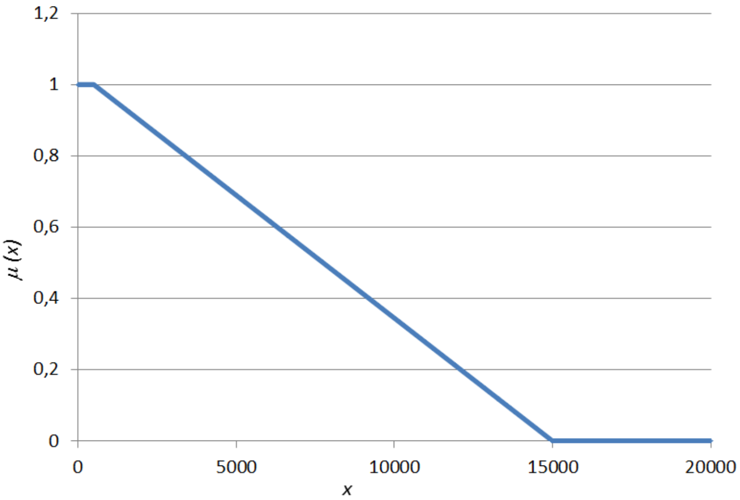
$$\mu(x) = \begin{cases} 0 & \text{if} \quad x < 4000 \\ 1 & \text{if} \quad 4000 \leq x < 20000 \\ 0 & \text{if} \quad x \geq 20000 \end{cases}$$

Fig. 5. Membership function – roads

Historical and cultural monuments (Hm) (data RA, distance, meter)

Map Algebra – Raster calculator

$$\mu(x) = \begin{cases} 1 & \text{if} \quad x < 500 \\ \frac{15000-x}{14500} & \text{if} \quad 500 \leq x \leq 15000 \\ 0 & \text{if} \quad x > 15000 \end{cases} \qquad \text{(Fig. 6.)}$$



for crisp sets

$$\mu(x) = \begin{cases} 1 & \text{if} \quad x \leq 7000 \\ 0 & \text{if} \quad x > 2000 \end{cases}$$

Fig. 6. Membership function - monuments

Bike trails (Bt) – net density (data RA)

Density – Kernel Density (radius 3 km), normalization

Hiking trails (Ht) – net density (data RA)

Density – Kernel Density (radius 3 km), normalization

Using ModelBuilder

The logical model of raster analysis is created in ModelBulder. In the preliminary stage the necessary data are collected and then union, selection, buffering and density are executed. The vector data are converted to raster data. If it is possible, the data reclassification and eventually euclidean distance are computed. Finally we can implement the raster operations to progressive evaluation of result according to desired expression written as

$$Ea \cap Fa \cap Bv \cap Ro \cap Hm \cap (Bt \cup Ht)$$

Our intent of this spatial model was to represent a base map for analysis of tourism regions in South Moravia. Final fuzzy map (Fig. 8.) depends on the expression and description of the problem, determination of particular criteria and assignment of the degree of membership to fuzzy sets. It is important to specify operations on fuzzy sets (in this case standard intersection and union). The data quality is essential.

Conclusion

This multicriteria decision analysis carries the obtaining of the basic spatial placement of significant tourist areas. It provides the results determined to compare with statistical data such as the attendance and the attractiveness of region, the substantiation of the low attendance in some places, finding their developing, linkup to public services and the direct consequences for hiking and bike trails.

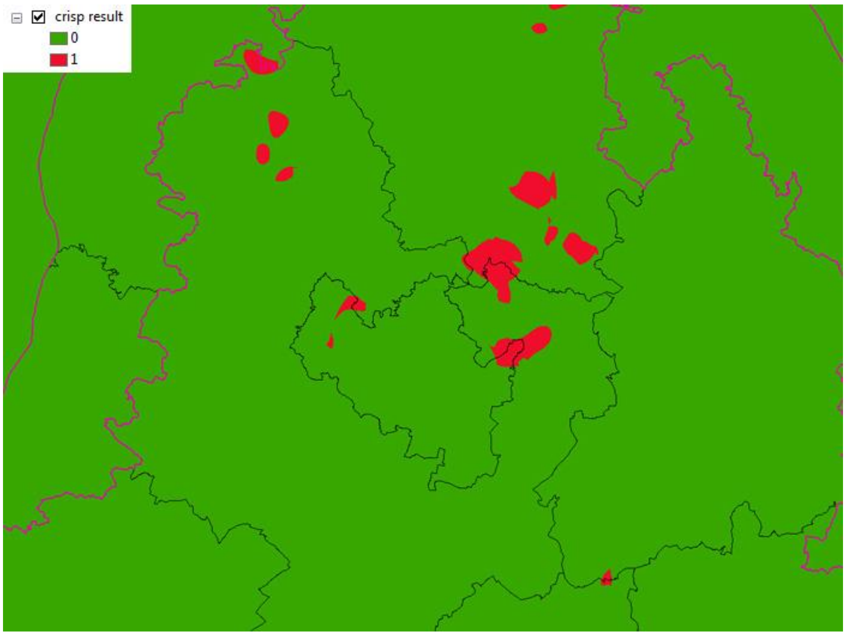


Fig. 7. The result of analysis near Brno and surrounding - crisp sets

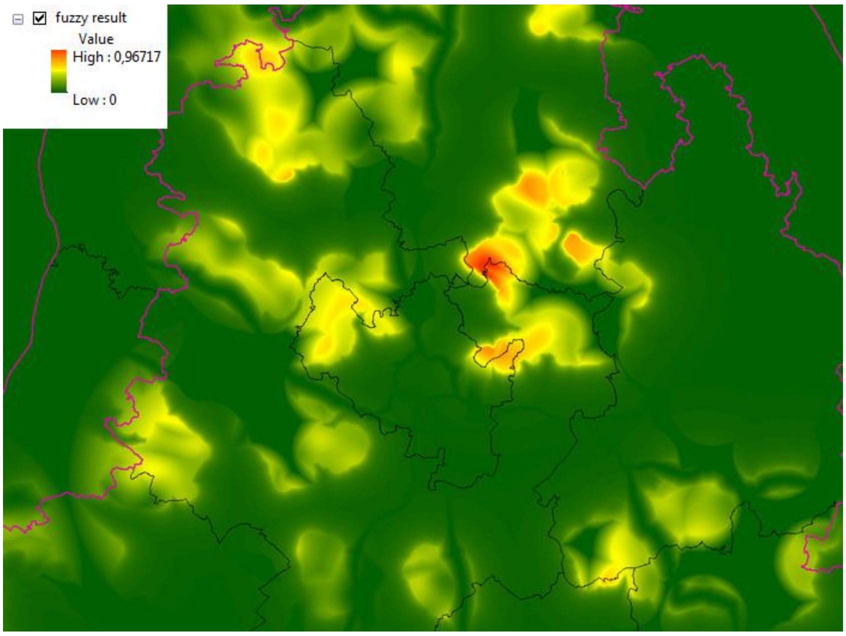


Fig. 8. The result of analysis near Brno and surrounding - fuzzy sets

SOLUTION OF BIKE TRAIL DIFFICULTY RATING

When we think about cycle route, we consider time, weather, trail length, points of interest and we solve the route difficultness or demandingness, too. We find out the route quality and suitability for different categories of target bikers. Bike trail difficulty recognizes whether the route is suitable for families with children, for recreational sportsmen, maybe for athletes. In 2003 and 2005 the projects were made with intent to collect information about cycle routes and their facilities. In 2007 the data were updated by terrain research, especially the status of surface and difficulty of bike trails.

During actual checking well-known routes it was verified that the characteristic of bike trail difficulty has already disagreed with the reality. Each rating depends on time, it is affected by the subjective view and data collection is a hard task in terrain. Therefore, we need to utilize another approach for instance by fuzzy reasoning. The slope and the quality or the type of the road surface which were chosen as analytical inputs impact on the difficulty.

The data are published on the web cycling portal of the South Moravian Region http://www.cyklo-jizni-morava.cz, including the interactive bike trail map with choosing routes and points of interest.

The modelling is accomplished over rasters in ArcGIS 10.1 using ModelBuilder and geoprocessing tools, especially Spatial Analyst Tools – Fuzzy Membership, Fuzzy Overlay, Raster Calculator, Cell Statistics and Python.

Methods

We use two input variables, X_1 for the type of the road surface and X_2 for the angle of the slope (both defined by crisp values) and output variable Y for the bike trail difficulty.

Assume the following input and output fuzzy subsets which are given by verbal values and rules representing their relationship.

Type of road surface (data CEDA StreetNet 2012)

K_1 - paved roads (asphalt, pavement, concrete)

K_2 - maintained roads (unpaved, gravel)

K_3 - other unpaved roads (forest and cart roads)

Angle of slope (DMT 2012, in degrees, cells size 10 meters)

S_1 - moderate slope

S_2 - steep slope

Bike trail difficulty

D_1 - small difficulty – easy difficult roads (suitable for families with children)

D_2 - intermediate difficulty – more difficult roads (suitable for recreational sportsmen)

D_3 - hard difficulty - very difficult roads (suitable for athletes)

Rules

P_1: if X_1 is K_1 and X_2 is S_1, then Y is D_1

P_2: if X_1 is K_2 and X_2 is S_1, then Y is D_1

P_3: if X_1 is K_3 and X_2 is S_1, then Y is D_2

P_4: if X_1 is K_1 and X_2 is S_2, then Y is D_2

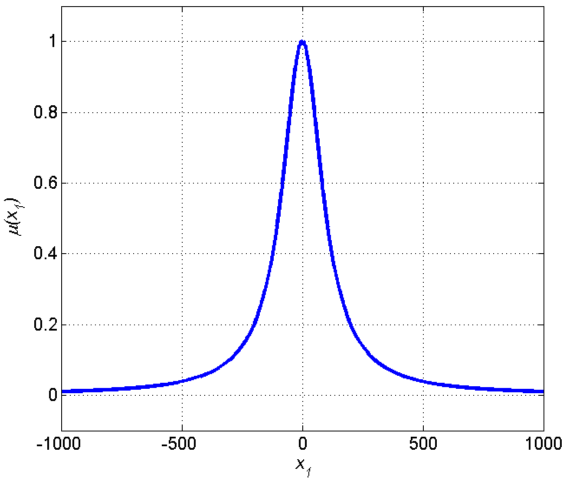
P_5: if X_1 is K_2 and X_2 is S_2, then Y is D_3

P_6: if X_1 is K_3 and X_2 is S_2, then Y is D_3

Observing X_1 is K' and X_2 is S'

Conclusion Y is D'

The fuzzy sets K_1, K_2, K_3 were given by the bell-shaped membership function Near (Midpoint 0, Spread 0,0001) available in the geoprocessing tools of ArcMap in the category Fuzzy Membership (Fig. 9.). The function expresses the close localization of the road as a fuzzy line [4] in network of roads (x_1 - distance from the road in meters).



$$\mu(x_1) = \frac{1}{1 + 0{,}0001\, x_1^{\,2}}$$

Fig. 9. Membership function for roads

Next figures show settings that define S_1, S_2 and D_1, D_2, D_3 (Fig. 10. and Fig. 11.)

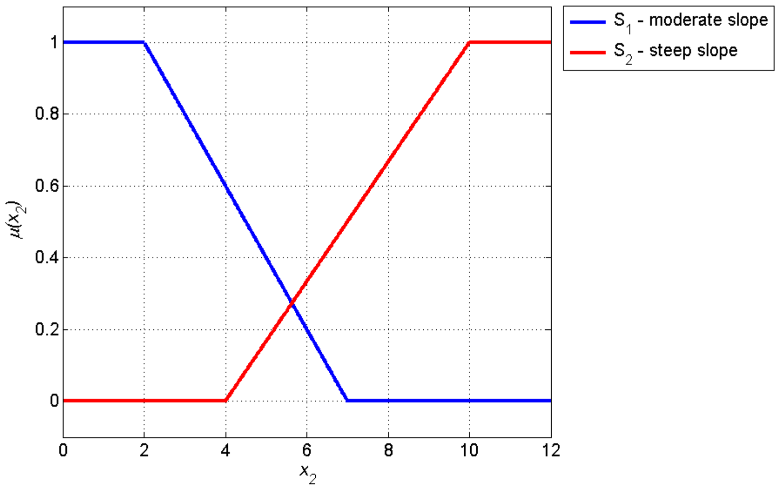


Fig. 10. Membership function for slope

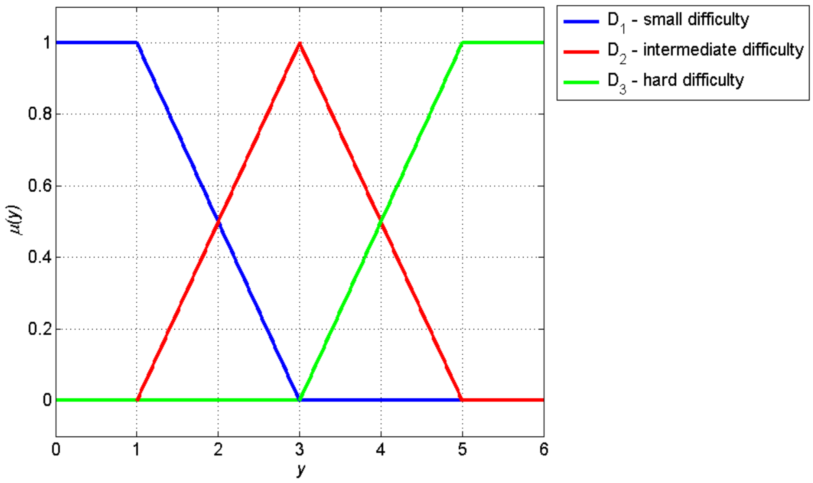


Fig. 11. Membership function for difficulty of road

I applied and compared several regulators and defuzzification methods and did the interpretation rule-by-rule. We declare w_j as the total weight of the j-th rule worked from particular weights of premises (roads, slope) w_{1j}, w_{2j}. The membership function of conclusion of the j-th rule is written $\mu_{D_j'}(y)$. This is the most important applied methods.

Mamdani's method (COS-TM-TM, COM-TM-TM)

$$\mu_{D'}(y) = \max_{j=1}^{k} \ T_M\left(T_M\left(w_{1j}, w_{2j}\right), \mu_{D_j}(y)\right) = \max_{j=1}^{k} \ \min\left(\min\left(w_{1j}, w_{2j}\right), \mu_{D_j}(y)\right)$$

Larsen's method (COS-TP-TM)

$$\mu_{D'}(y) = \max_{j=1}^{k} \ T_P\left(T_M\left(w_{1j}, w_{2j}\right), \mu_{D_j}(y)\right) = \max_{j=1}^{k} \ \left(\min\left(w_{1j}, w_{2j}\right) \cdot \mu_{D_j}(y)\right)$$

Mamdani's method (COS-TM-TM)

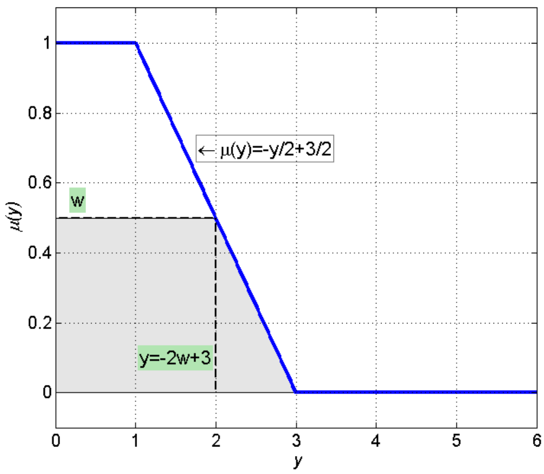
Considering evaluation of the road surface and reasoning of conclusion rule-by-rule, we will choose (COS-TM-TM) the centroid of sums which means calculation.

$$y_{D_j'}^{CoS} = \frac{\sum\limits_{1 \le j \le 6} \left(\int\limits_{Y} \mu_{D_j'}(y)\, y\, dy \right)}{\sum\limits_{1 \le j \le 6} \left(\int\limits_{Y} \mu_{D_j'}(y)\, dy \right)} =$$

$$= \frac{\int\limits_{Y} \mu_{D_1'}(y)\, y\, dy + \int\limits_{Y} \mu_{D_2'}(y)\, y\, dy + \int\limits_{Y} \mu_{D_3'}(y)\, y\, dy + \int\limits_{Y} \mu_{D_4'}(y)\, y\, dy + \int\limits_{Y} \mu_{D_5'}(y)\, y\, dy + \int\limits_{Y} \mu_{D_6'}(y)\, y\, dy}{\int\limits_{Y} \mu_{D_1'}(y)\, dy + \int\limits_{Y} \mu_{D_2'}(y)\, dy + \int\limits_{Y} \mu_{D_3'}(y)\, dy + \int\limits_{Y} \mu_{D_4'}(y)\, dy + \int\limits_{Y} \mu_{D_5'}(y)\, dy + \int\limits_{Y} \mu_{D_6'}(y)\, dy}$$

The total weight of the j-th rule w_j is the minimum of the particular weights of the premises (roads, slope) w_{1j}, w_{2j} in this rule (simply signed w). The membership function of the conclusion of the j-th rule is presented as $\mu_{D_j'}(y) = \min\left(w_j, \mu_{D_j}(y)\right)$. The membership $\mu_{D_j}(y)$ is simply denoted $\mu(y)$.

In the first and the second rule we evaluate small difficulty D_1 (Fig. 12).



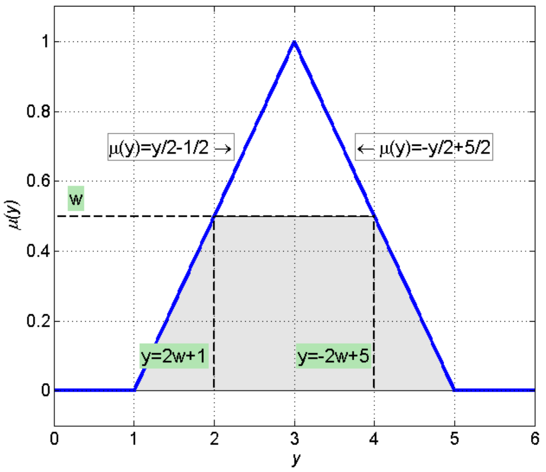
$$\int_{0}^{-2w+3} w\, y\, dy + \int_{-2w+3}^{3} \left(-\frac{y}{2} + \frac{3}{2} \right) y\, dy = \underline{\frac{2}{3} w^3 - 3w^2 + \frac{9}{2} w}$$

and

$$\int_{0}^{-2w+3} w\, dy + \int_{-2w+3}^{3} \left(-\frac{y}{2} + \frac{3}{2} \right) dy = \underline{-w^2 + 3w}$$

Fig. 12. Membership function for small difficulty

In the third and the fourth rule we evaluate intermediate difficulty D_2 (Fig. 13).



$$\int_1^{2w+1}\left(\frac{y}{2}-\frac{1}{2}\right) y\,dy + \int_{2w+1}^{-2w+5} w\,y\,dy + $$

$$+\int_{-2w+5}^{5}\left(-\frac{y}{2}+\frac{5}{2}\right) y\,dy = \underline{-6w^2+12w}$$

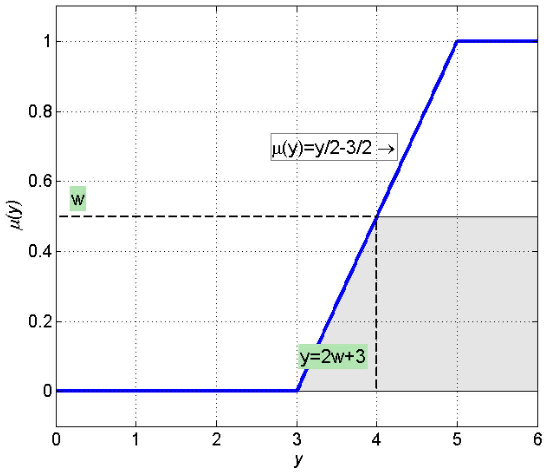
and

$$\int_1^{2w+1}\left(\frac{y}{2}-\frac{1}{2}\right) dy + \int_{2w+1}^{-2w+5} w\,dy + $$

$$+\int_{-2w+5}^{5}\left(-\frac{y}{2}+\frac{5}{2}\right) dy = \underline{-2w^2+4w}$$

Fig. 13. Membership function for intermediate difficulty

In the fifth and the sixth rule we evaluate hard difficulty D_3 (Fig. 14).



$$\int_3^{2w+3}\left(\frac{y}{2}-\frac{3}{2}\right) y\,dy + \int_{2w+3}^{6} w\,y\,dy = \underline{-\frac{2}{3}w^3-3w^2+\frac{27}{2}}$$

and

$$\int_3^{2w+3}\left(\frac{y}{2}-\frac{3}{2}\right) dy + \int_{2w+3}^{6} w\,dy = \underline{-w^2+3w}$$

Fig. 14. Membership function for hard difficulty

Mamdani's method (COM-TM-TM)

We evaluate by the centroid of singletons Center of Maximum (COM-TM-TM) using the mean of the maximum.

$$y_{D_j'}^{CoM} = \frac{\sum_{j=1}^{k} y_j \cdot \mu_{D_j'}(y_j)}{\sum_{j=1}^{k} \mu_{D_j'}(y_j)} =$$

$$= \frac{y_1 \cdot \mu_{D_1'}(y_1) + y_2 \cdot \mu_{D_2'}(y_2) + y_3 \cdot \mu_{D_3'}(y_3) + y_4 \cdot \mu_{D_4'}(y_4) + y_5 \cdot \mu_{D_5'}(y_5) + y_6 \cdot \mu_{D_6'}(y_6)}{\mu_{D_1'}(y_1) + \mu_{D_2'}(y_2) + \mu_{D_3'}(y_3) + \mu_{D_4'}(y_4) + \mu_{D_5'}(y_5) + \mu_{D_6'}(y_6)} =$$

By substituting values:

$$= \frac{\frac{0-2w_1+3}{2} \cdot w_1 + \frac{0-2w_2+3}{2} \cdot w_2 + \frac{2w_3+1-2w_3+5}{2} \cdot w_3 + \frac{2w_4+1-2w_4+5}{2} \cdot w_4 +}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6}$$

$$\frac{+ \frac{2w_5+3+6}{2} \cdot w_5 + \frac{2w_6+3+6}{2} \cdot w_6}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6} = \frac{-w_1^2 + \frac{3}{2} w_1 - w_2^2 + \frac{3}{2} w_2 + 3 \cdot w_3 + 3 \cdot w_4 + w_5^2 + \frac{9}{2} w_5 + w_6^2 + \frac{9}{2} w_6}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6}$$

The models of Mamdani's methods in ModelBulder are shown in Fig. 15.

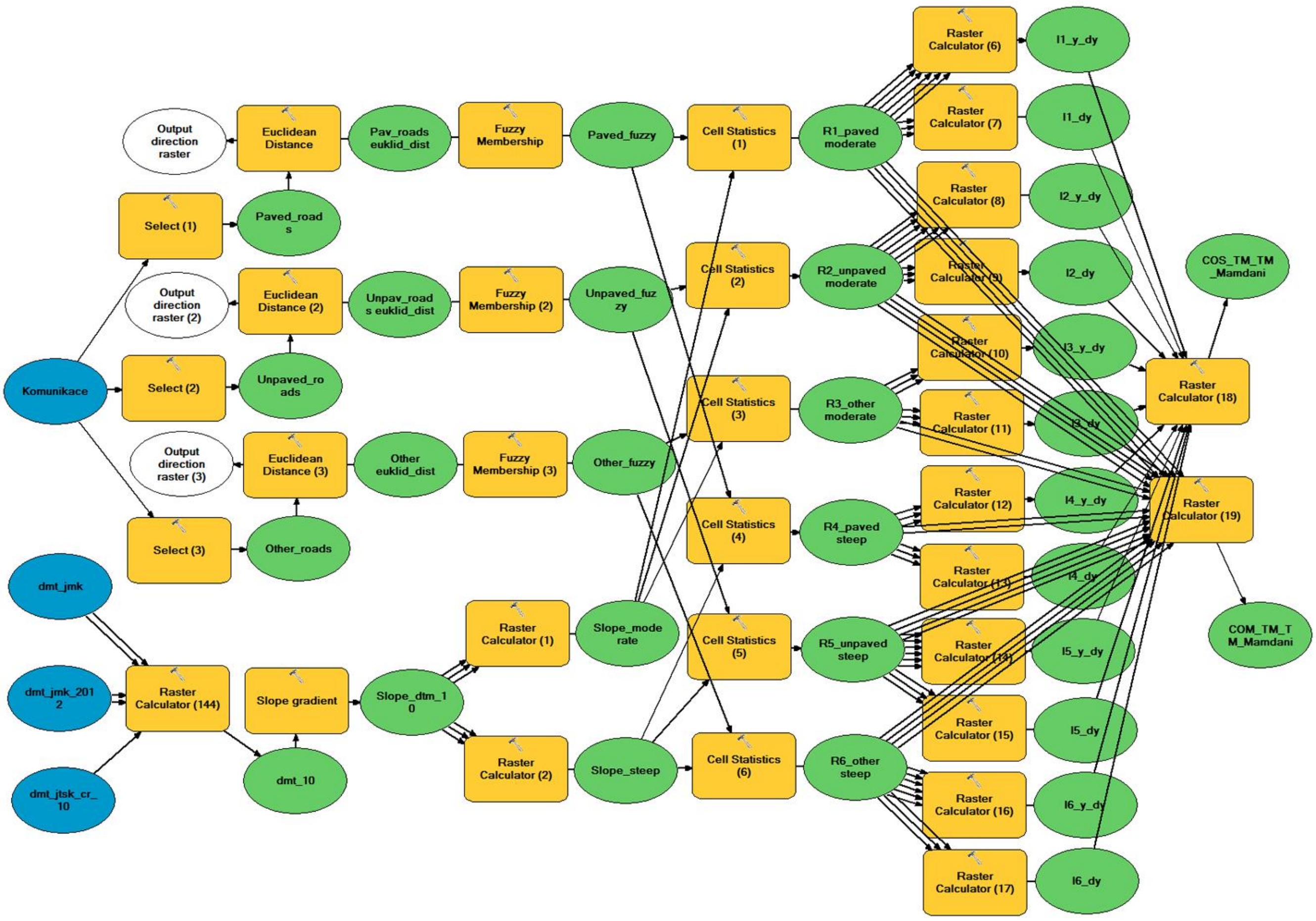


Fig. 15. The models of Mamdani's methods (COS-TM-TM, COM-TM-TM)

Comparison of the methods and defuzzification

The data of well-known parts of the bike trails which were possible to classify were selected to compare the best methods. We can use the comparison of the maximum, minimum, arithmetic mean and standard deviation according to the difficulty of the bike trails, then we monitored frequency histograms.

Mamdani's method is well representative with defuzzification CoS but also with defuzzification CoM, where there is the bigger value range and the higher frequency on the intervals of the maximum occurrence.

Larsen's method has similar characteristics. However, it is not suitable for the bike trails with the intermediate difficulty and highlights the bike trails with the small and hard difficulty.

The table compares the percentage of the bike trails suitable for the membership in interval $\langle 0{,}25; 1\rangle$ according to small, intermediate and hard difficulty having regard to their whole choice for the individual difficulties and the choosing methods.

Table 1. Part of suitable bike trails for membership $\langle 0{,}25; 1\rangle$

difficulty	**small**	**intermediate**	**hard**	**all**
COS-TM-TM	96,6 %	83,7 %	73,7 %	**84,1 %**
COM-TM-TM	97,1 %	75,0 %	74,5 %	76,0 %
COS-TP-TM	97,1 %	67,9 %	74,8 %	69,4 %

The sum value of the percentages expresses the precision of the individual method. We can see that Mamdani's method bluntly dominates, especially with defuzzification CoS.

The results of Larsen's method are quite good. This method is not much reliable in the evaluation of the intermediate difficult bike trails. It significantly competes with Mamdani's method within the small and hard difficult bike trails. These methods are compared at the selected region (Fig. 16), where Mamdani's method with defuzzification CoM is characterized by big differences and Larsen's method by small differences of values of the bike difficulty. Mamdani's method with defuzzification CoS is the most reliable.

We choose the bike trail difficulty obtained by Mamdani's method with the defuzzification CoS for another analytical processing. It will permit to reclassify the attribute of the current bike difficulty and to add the difficulty of the other roads for the routing as the finding of the optimal road according to the difficulty.

Making use of Mamdani's raster to finding the bike trail rating

We assume that Mamdani's raster with defuzzification CoS is the main input to our analysis. The aim of this way is rating of all roads, not only cycle trails, registered in street net feature class.

First, we perform the extract of the cells of this raster that corresponds to roads defined by a buffer mask. Then the raster is converted to point feature class with the attribute from raster. The aspect as the slope direction is added to every point which obtains the near information about distance to the nearest road line, too. Finally, we extract and join the value of the difference Mamdani's raster and Mamdani's "null" raster which doesn´t consider the altitude. This difference influences the increase of the road difficulty compared to the flat surface. These steps are implemented by ModelBuilder and point feature class is the result – feature class called Points_join_Mam_aspect_minus in Fig. 17.

The next part takes open-source programming language Python that is both powerful and easy to learn. Particular steps are reflected in the following scheme (Fig. 17) where these components are used.

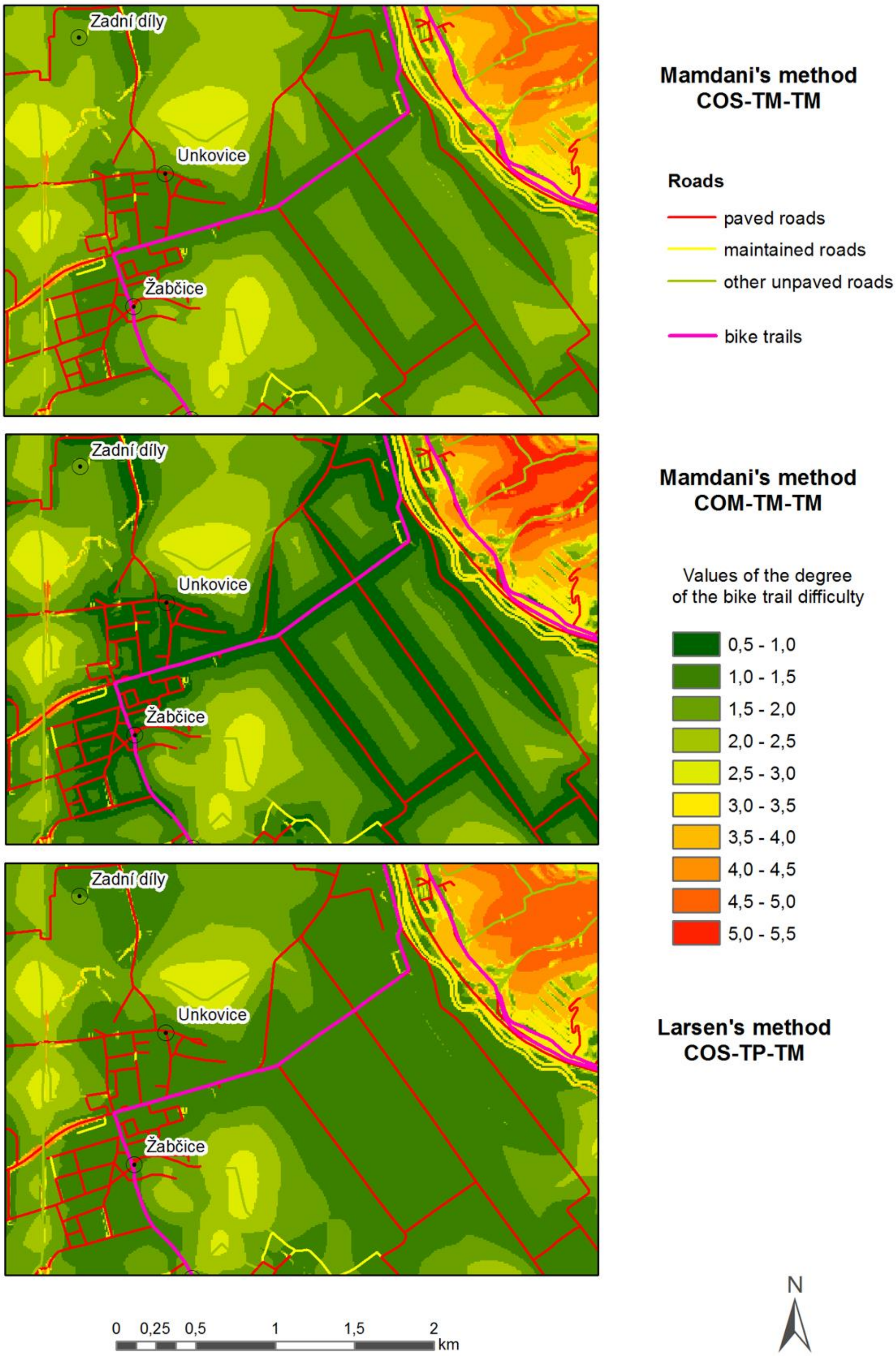


Fig. 16. Comparison of the methods in the region detail

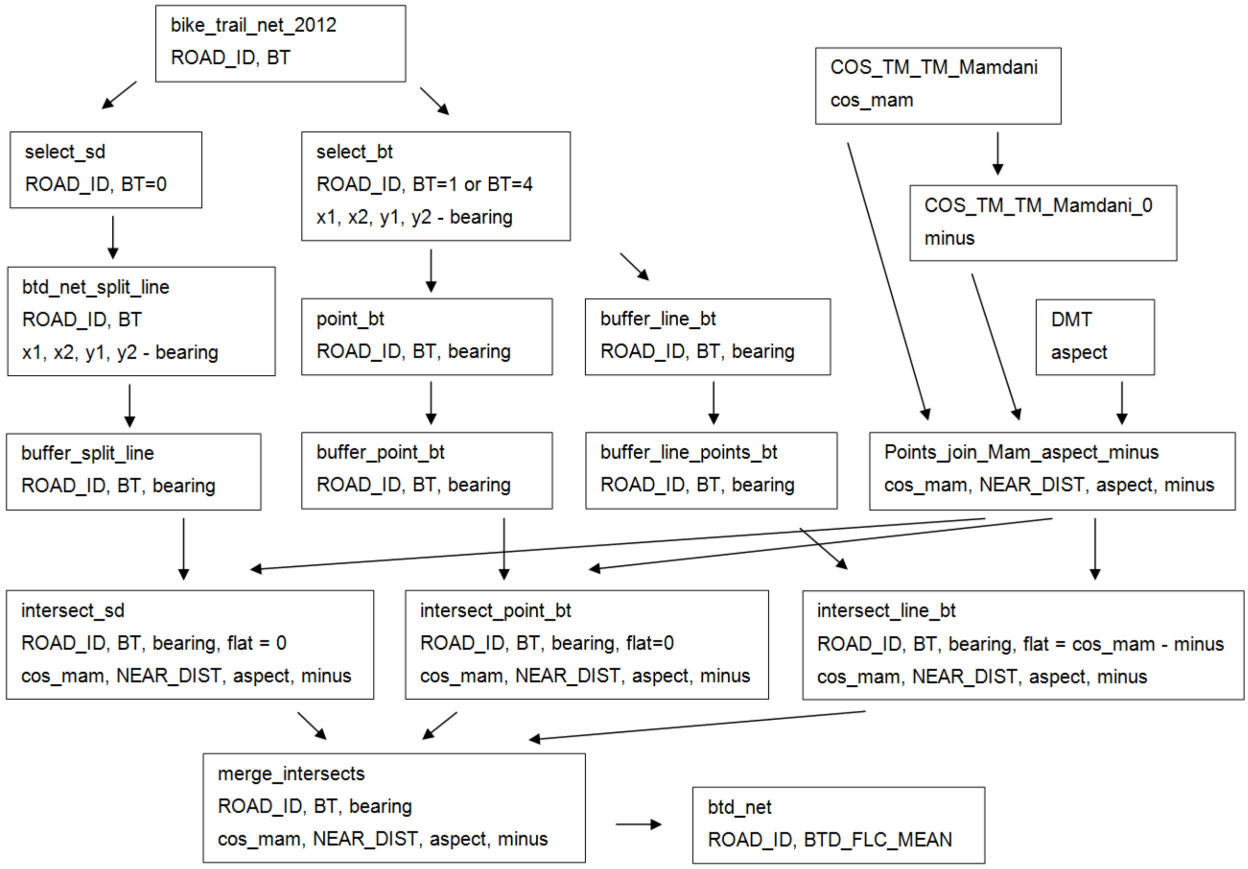


Fig. 17. Scheme for analysis by Python – layers and their important attributes

Explanatory text:

bike_trail_net_2012 – input feature class involving street net and bike trail sections with actual fields

select_sd – selection of bike trails except bridges and tunnels

btd_net_split_line – seperation of bike trail sections to line segments, new geometry and completion bearing attribute (azimuth)

buffer_split_line – 10 meters buffer of the line segments, bearing (line azimuth)

select_bt - selection of bridges and tunnels

point_bt - beginnings and endings of bridges and tunnels

buffer_point_bt – 10 meters buffer of endpoints of bridges and tunnels

buffer_line_bt - 10 buffer of line of bridges and tunnels

buffer_line_points_bt – symmetrical difference buffer_line_bt and buffer_point_bt

intersect_sd – intersect buffer_split_line and Points_join_Mam_aspect_minus

intersect_point_bt – intersect buffer_point_bt and Points_join_Mam_aspect_minus

intersect_line_bt – intersect buffer_line_points_bt and Points_join_Mam_aspect_minus

merge_intersects – merge of all intersects

btd_net – output bike trail sections with assignment of new bike trail difficulty

The most important step by Python creates btd table for joining to btd_net feature class with new bike trail difficulty field. Initial testing script worked with all points from feature class merge_intersects and calculated only arithmetic mean of particular roads according to road id. Because the points are in the different distance from road line it is more accurate to think about weighted average in the context of fuzzy membership of points. In the distance 10 meters is membership 0 and on the line is membership 1 (BTD_FUZZY_MEAN).

In this approach we can see the biggest problem in the line direction compared with the slope direction. Bike trail sections in the direction of the contour line have a hard difficulty. And that also brought me to the fact that it will be useful to take the current difficulty from Mamdani's raster for the line segments in direction slope line and use difficulty from Mamdani's "null" raster (flat surface) for the segments in direction contour line. Consequently, the difficulty depends on the difference these rasters and the angle between the slope direction and the road line azimuth. The product of this deviation of straight lines and the ninetieth of this difference means the reduction of difficulty (BTD_FCL_MEAN).

Finally, I tried to reduce mistakes for short road sections and roads in bridges and at tunnels. The difficulty of the bridges and tunnels road is specified mainly by points at their beginning and ending. The points along the line are reduced to the flat value (Fig. 19).

Conclusion

The bike trail difficulty is important readout for planning routes of bike tours. Mainly, it depends on the quality of the road surface and the slope. We can express the requests for the bike trail difficulty fairly verbally by rules that are processed using the fuzzy sets and the compositional rule of inference and Mamdani's method. This method has reached the best effect with the defuzzification the centroid of sums and using the integral calculus.

The main aim of this paper is the exploitation and map presentation of the results on the web cycling portal of the South Moravian Region http://www.cyklo-jizni-morava.cz/. The analysis extends the difficulty of the bike trails to all roads. Considering fuzzy approach we can imagine the region compactly as a whole of the seamless bike trail difficulty raster fuzzy map and as the map of bike trail difficulty rating. The reclassification of the current difficulty and the update of the road difficulty network are very important to the improvement of the bike routing depending on the required target group (family with children, recreational sportsman and athlete).

There are some inaccuracies and problems especially on the connections of the miscellaneous types of roads. I have some ideas to improve the actual bike rating and remove the found mistakes. It is the opportunity to use more rules in Mamdani's method or eventually add other linguistic variables such as elevation and road length.

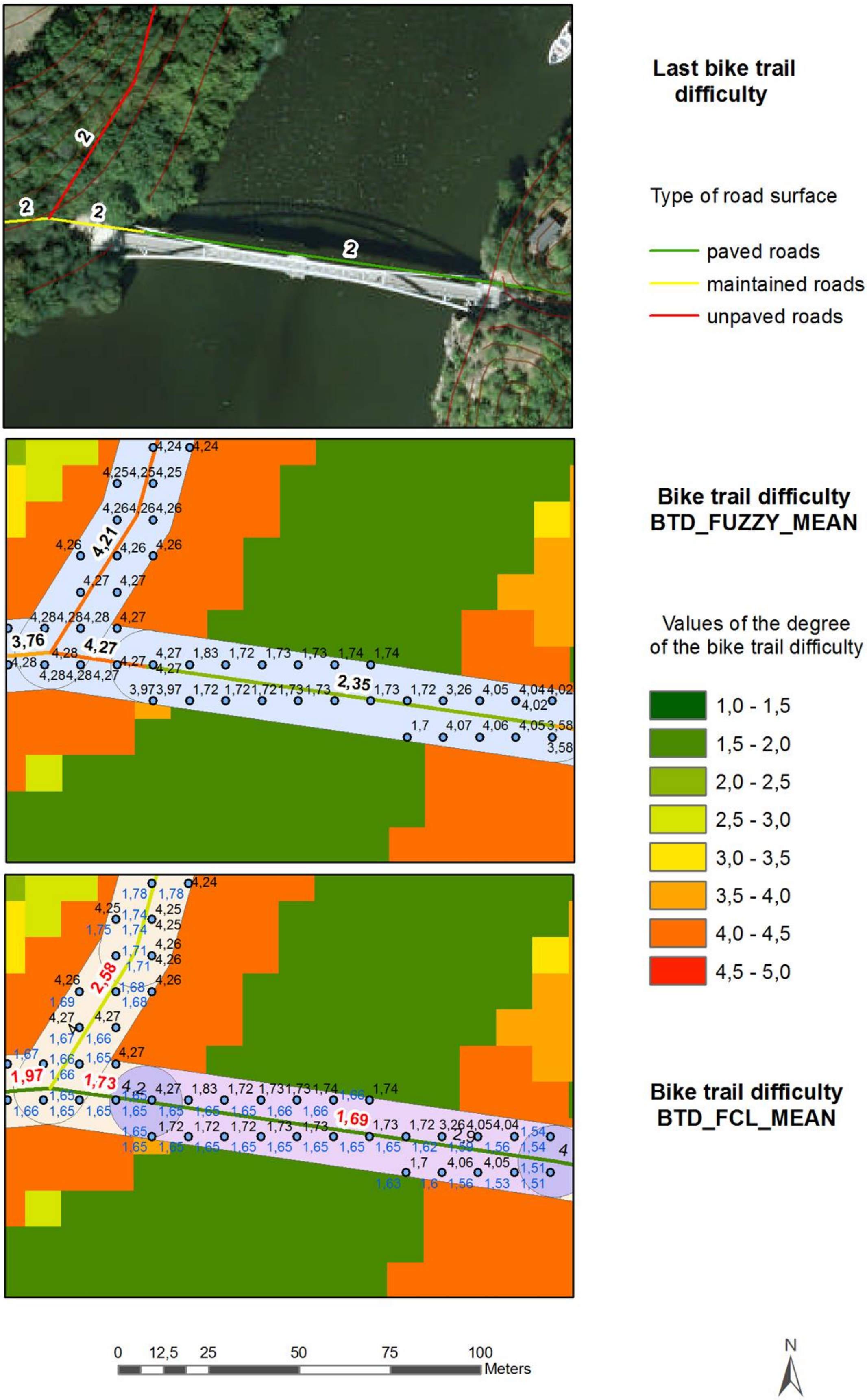


Fig. 19. Comparison of the last bike trail rating, bike trail rating BTD_FUZZY_MEAN and BTD_FCL_MEAN

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