TRAIN PLATFORMING PROBLEM

Ľudmila JÁNOŠÍKOVÁ[1], Michal KREMPL[2]

[1] Department of Transportation Networks, Faculty of Management Science and Informatics, University of Žilina, Univerzitná 1, 010 26 Žilina, Slovak Republic

Ludmila.Janosikova@fri.uniza.sk

[2] Institute of Transport, Faculty of Mechanical Engineering, VŠB Technical University of Ostrava, 17. listopadu 15, 708 33 Ostrava - Poruba, Czech Republic

Michal.Krempl.st@vsb.cz

Abstract

The train platforming problem consists in the allocation of passenger trains to platforms in a railway station. One of the important problems a dispatcher has to solve, especially in a large railway station, is to decide, at which platform track an approaching train should arrive. There is a tool helping him in his job called the track occupancy plan. The plan specifies for each arriving or departing train the platform track along with the time slot during which the track will be occupied by the train. This paper deals with a method for computer-aided design of the track occupancy plan. The problem is formulated as a bi-criterion mixed integer programming problem. The first objective is to minimise the deviations of the arrival and departure times proposed by the model from the times specified by the timetable. The second criterion maximises the desirability of the platform tracks to be assigned to the trains. The model is solved using a lexicographic approach and the local branching algorithm. The model was verified by using the real data of Prague main station. Results of the experiments are included.

Keywords: train platforming, scheduling, mixed integer mathematical programming; multiple-objective programming

1. INTRODUCTION

The train platforming problem is a subproblem of the generation of a timetable for a railway company. The generation of a timetable is a hierarchical process. At the first stage, a preliminary timetable for the whole network is proposed. In this phase, a macroscopic viewpoint at the railway network is applied. Stations are considered as black boxes. Capacity limits of particular stations and the movement of trains inside the stations are not taken into account. Then, at the second stage, a microscopic viewpoint related to stations is applied. At every station, the network timetable is checked whether it is feasible with respect to capacity, safety and train operators' preferences. This process results in a track occupancy plan which specifies for each arriving or departing train the platform track along with the time slot during which the track will be occupied by the train. Cargo trains do not affect the plan since they travel mostly in night, when there are fewer passenger trains, they use different tracks in the station, and in case of conflicting movements they can wait at the entry signal.

In the Czech and Slovak Republic, planning train movements through the station is done by hand, using planner's experience and a set of rules determined by a railway company. The main goal of this research is to design a more sophisticated approach which would serve as a planner's decision supporting tool and result in a better track occupancy plan. Improvement in the plan quality results in:

1. better management of train operation in the station, namely:
   a) shorter times of routes occupation by arriving and departing trains,
   b) uniform workload of the infrastructure elements, such as tracks, switches, and platforms, which leads to a more robust plan resistant to random disturbances;
2. higher service quality perceived by passengers, namely:
   a) shorter distances needed for changing trains,

b) more appropriate platforms (platforms near to ticket sales points and to the station entrance, platforms equipped by station shops or catering etc.),

c) less probability of changing the planned platform when the train delays.

3. meeting train operators' requirements on arrival and departure times and platforms assigned to trains.

Routing and scheduling trains at a station has been studied by researchers in countries, where large, busy stations with capacity constraints can be found. Billionet (2003) addresses only the routing problem. The problem is modelled using a graph theory and the integer programming formulation of the resulting graph colouring problem is solved. However, the *k* colouring problem is not indeed an optimisation problem, it means any feasible solution is acceptable and the problem formulation does not reflect the solution quality, such as route lengths or platform preferences for individual trains. Zwaneveld (1997) and Zwaneveld, Kroon, and van Hoesel (2001) formulate the problem of train routing as a weighted node packing problem, using bivalent programming, while the solution algorithm applies the branch-and-cut method. A disadvantage of the above presented models is that the calculations connected with them are computationally too complex and time consuming. Another, practically oriented approach has given up on applying the integer programming methods, and replaced them by the heuristics, solving the scheduling and routing problems at a time (Carey and Carville, 2003). The algorithm incorporates, or considers, the operational rules, costs, preferences and trade-offs, which are applied by experts creating plans manually. The shortcoming of this approach is obvious: since it is a heuristics, the optimality of the resulting plan is not guaranteed.

Other way of research, e.g. (Bažant and Kavička, 2009, Chakroborty and Vikram, 2008), has been directed at operational train management. In real time it is necessary to reflect the requirements of the operation burdened with irregularities, i.e. to re-schedule the arrivals and departures times, and/or re-route trains.

In this paper we propose a mixed integer programming (MIP), bi-criteria model of the train platforming problem. The problem can be solved by a lexicographic approach, where particular criteria are ranked according to their importance.

2. PROBLEM FORMULATION

The train platforming problem consists of the following partial issues. For each train,

- a platform track must be specified at which the train should arrive; the platform track assignment determines the route, on which the train approaches,
- arrival time at the platform and departure time from the platform need to be determined.

The solution should minimise deviations from the planned arrival and departure times and maximise the total preferences for platforms and routes.

The inputs to the mathematical programming model are as follow:

1. track layout of the station, which is necessary for determining feasible platform tracks for a train and conflicting routes,
2. list of trains, where the data required for each train include:
   a) planned time of its arrival at the platform,
   b) planned time of its departure from the platform,
   c) line on which the train arrives (in-line) and departs (out-line),
   d) list of feasible platform tracks with their desirability for the train,
   e) category of the train.

All time data are given in minutes.

Further on we present the formulation of the MIP model. First we need to explain the symbols used:

Subscripts which in the mathematical model represent objects

i, i', j train

k, k' platform track

Input parameters (constants)

t_i^{Pa} planned arrival time of train i at the platform

t_i^{Pd} planned departure time of train i

t^{Cn} standard amount of time passengers take to change trains (depends on particular railway station)

I_i arrival line track (in-line) for train i

O_i departure line track (out-line) for train i

c_i category of train i; $c_i = 1$ for regional stopping trains and increases with the speed and distance travelled by the train. We have to divide train into categories, because international fast express trains have obviously higher importance than the regional ones. Delays of international trains can commit more traffic problems and extra costs than delays of regional trains.

t^{min} minimum dwell time of a train at the platform

t^{max} maximum time interval, in which two train movements are tested for a conflict

p_{ik} preference coefficient; it reflects the desirability of the assignment of platform track k to train i

s_{ik} number of switches on the route of train i from the arrival line track to platform track k and from platform track k to the departure line track

$s_i^{\min}$ number of switches on the shortest train route in the station

$s_i^{\max}$ number of switches on the longest train route in the station

$a(l,k,l',k')$ coefficient, which has value *true*, if the route connecting line l to platform track k conflicts with the route connecting line l' to platform track k'; if there exists any route connecting line l to track k and any route connecting line l' to track k' such that these two routes do not conflict, then $a(l,k,l',k')$ = false. If both trains use the same station or line tracks (i.e. $k = k'$ or $l = l'$), then $a(l,k,l',k')$ = true. The existence of route conflicts can be identified in advance from a detailed map of the track layout.

We adopted the concept of conflicting routes and conflict solving from Carey and Carville (2003). If two trains are on conflicting routes we must ensure that there is at least a required minimum headway (time interval) between them, for safety and signalling reasons. For example, let $h(i,k,i',k')^{da}$ be the minimum headway required between train i departing from track k and the next train i' arriving at track k'. The superscripts d and a denote departure and arrival, and the order of the superscripts indicates the order of the trains, i.e., train i is followed by i'. Similarly we have $h(i,k,i',k')^{aa}$, $h(i,k,i',k')^{ad}$ and $h(i,k,i',k')^{dd}$ for combinations arrival – arrival, arrival – departure, departure – departure. We need not introduce subscripts to denote the in-lines or out-lines used by trains since for an arriving train i the in-line is already specified by I_i, and for a departing train i the out-line is specified by O_i.

The preference coefficient p_{ik} may reflect:

- operator's preferences of platforms,
- the distance of the track k to the connecting trains,
- the length of the route used by train i arriving to or departing from platform track k. The smoother and shorter the route is, the less the possibility of a conflict with other trains is, hence the probability of delay propagation decreases.

In our model, coefficient p_{ik} is set according to the following formula:

$$p_{ik} = \begin{cases} 1 & \text{if track } k \text{ is the planned (or desired) track for train } i \\ 0.9 & \text{if track } k \text{ is located at the same platform as the planned track} \\ 0.8\left(\dfrac{s_i^{\max} - s_{ik}}{s_i^{\max} - s_i^{\min}}\right) & \text{otherwise} \end{cases}$$

Sets of objects

K set of all platform tracks

K(*i*) set of feasible platform tracks for train *i*

U set of all arriving, departing, and transit trains

W(j) set of all connecting trains, which has to wait for train *j*

$V^{aa} = \{(i,j) : i,j \in U, i < j, \left|t_i^{Pa} - t_j^{Pa}\right| \le t^{\max}\}$ set of ordered pairs of those trains that may arrive concurrently

$V^{ad} = \{(i,j) : i,j \in U, i < j, \left|t_i^{Pa} - t_j^{Pd}\right| \le t^{\max}\}$ set of ordered pairs of those trains that arriving train *i* and departing train *j* may travel concurrently

$V^{da} = \{(i,j) : i,j \in U, i < j, \left|t_i^{Pd} - t_j^{Pa}\right| \le t^{\max}\}$ set of ordered pairs of those trains that departing train *i* and arriving train *j* may travel concurrently

$V^{dd} = \{(i,j) : i,j \in U, i < j, \left|t_i^{Pd} - t_j^{Pd}\right| \le t^{\max}\}$ set of ordered pairs of those trains that may depart concurrently

Decision and auxiliary variables of the model

$$\text{for } i \in U, k \in K(i) : x_{ik} = \begin{cases} 1 & \text{if track } k \text{ is assigned to train } i \\ 0 & \text{otherwise} \end{cases}$$

u_i difference between the planned and real arrival time of train *i* at a platform, $i \in U$

v_i difference between the planned and real departure time of train *i* from a platform, $i \in U$

The following auxiliary variables ***y*** are introduced for the couple of those trains *i* and *j* that may travel concurrently. They enable to express safety headways between conflicting trains.

$$\text{for } i,j \in U, i < j : y_{ij}^{aa} = \begin{cases} 1 & \text{if train } i \text{ arrives before train } j \text{ arrives} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for } (i,j) \in V^{ad} : y_{ij}^{ad} = \begin{cases} 1 & \text{if train } i \text{ arrives before train } j \text{ departs} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for } (i,j) \in V^{da} : y_{ij}^{da} = \begin{cases} 1 & \text{if train } i \text{ departs before train } j \text{ arrives} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for } (i,j) \in V^{dd} : y_{ij}^{dd} = \begin{cases} 1 & \text{if train } i \text{ departs before train } j \text{ departs} \\ 0 & \text{otherwise} \end{cases}$$

After these preliminaries, the mathematical model can be written as follows:

minimise $$c_i \sum_{i \in U} (u_i + v_i) \tag{1}$$

maximise $$\sum_{i \in U} \sum_{k \in K(i)} p_{ik} x_{ik} \tag{2}$$

subject to

$$v_i + t_i^{Pd} \geq u_i + t_i^{Pa} + t^{\min} \qquad \forall i \in U \tag{3}$$

$$v_i + t_i^{Pd} \geq u_j + t_j^{Pa} + t^{Cn} \qquad \forall j \in U; i \in W(j) \tag{4}$$

$$u_{i'} + t_{i'}^{Pa} \geq u_i + t_i^{Pa} + h(i,k,i',k')^{aa} - M\left(1 - y_{ii'}^{aa}\right) - M(1 - x_{ik}) - M(1 - x_{i'k'})$$
$$\forall (i,i') \in V^{aa}, k \in K(i), k' \in K(i') : a(I_i, k, I_{i'}, k') \tag{5}$$

$$u_i + t_i^{Pa} \geq u_{i'} + t_{i'}^{Pa} + h(i',k',i,k)^{aa} - My_{ii'}^{aa} - M(1 - x_{ik}) - M(1 - x_{i'k'})$$
$$\forall (i,i') \in V^{aa}, k \in K(i), k' \in K(i') : a(I_i, k, I_{i'}, k') \tag{6}$$

Constraints (7) – (12) are specified for the other combinations of arrival – departure and have similar meaning as (6).

$$u_i + t_i^{Pa} \geq v_{i'} + t_{i'}^{Pd} + h(i',k,i,k)^{da} - My_{ii'}^{aa} - M(1 - x_{ik}) - M(1 - x_{i'k})$$
$$\forall i, j \in U, i < j, k \in K(i) \cap K(i') \tag{13}$$

$$u_{i'} + t_{i'}^{Pa} \geq v_i + t_i^{Pd} + h(i,k,i',k)^{da} - M\left(1 - y_{ii'}^{aa}\right) - M(1 - x_{ik}) - M(1 - x_{i'k})$$
$$\forall i, j \in U, i < j, k \in K(i) \cap K(i') \tag{14}$$

$$y_{ij}^{aa} = 1 \quad \forall i, j \in U, i \neq j, I_i = I_j, t_i^{Pa} \leq t_j^{Pa} \tag{15}$$

$$\sum_{k \in K(i)} x_{ik} = 1 \qquad \forall i \in U \tag{16}$$

$$u_i, v_i \geq 0 \qquad \forall i \in U \tag{17}$$

$$x_{ik} \in \{0,1\} \qquad \forall i \in U \ \ \forall k \in K(i) \tag{18}$$

$$y_{ij}^{aa} \in \{0,1\} \qquad \forall i, j \in U, i < j \tag{19}$$

$$y_{ij}^{ad} \in \{0,1\} \qquad \forall (i,j) \in V^{ad} \tag{20}$$

$$y_{ij}^{da} \in \{0,1\} \qquad \forall (i,j) \in V^{da} \tag{21}$$

$$y_{ij}^{dd} \in \{0,1\} \qquad \forall (i,j) \in V^{dd} \tag{22}$$

Model description

Objective function (1) minimises the weighted deviations of the arrival and departure times proposed by the model from the times specified by the timetable. The weights cause that long-distance/high- speed trains will respect planned times and regional trains will be postponed if necessary. The second criterion maximises the desirability of the platform tracks to be assigned to the trains.

Constraint (3) ensures that a minimum dwell time needed for boarding and alighting must be kept.

Constraint (4) states that connecting train *i* with real departure $v_i + t_i^{Pd}$ has to wait in station to time at least $u_j + t_j^{Pa} + t^{Cn}$.

Constraints (5) – (12) ensure that a minimum headway will be kept between conflicting trains. More precisely, constraint (5) states that if trains *i* and *i′* have planned arrival times within t^{max} and train *i* arrives at platform track *k* before train *i′* arrives at track *k′*, i.e.

$$x_{ik} = 1, x_{i'k'} = 1, y_{ii'}^{aa} = 1, \qquad (\mathbf{23})$$

and trains are on conflicting routes (i.e. $a(l_i,k,l_{i'},k')$ is *true*), then train *i′* is allowed to arrive at least $h(i,k,i',k')^{aa}$ minutes later than train *i*. If at least one of the conditions (23) is not met (e.g. train *i* is not assigned to track *k*), then constraint (5) becomes irrelevant as the right-hand side is negative (*M* is a suitably picked high positive number). If train *i′* is followed by train *i* ($y_{ii'}^{aa} = 0$), then *i* is allowed to arrive at least $h(i',k',i,k)^{aa}$ minutes later than *i′*, which is ensured by constraint (6). Constraints (7) – (12) have a similar meaning for the other combinations of arrival – departure.

Constraints (13) – (14) ensure that a train will not be dispatched to an occupied track. If train *i′* is followed by train *i* ($y_{ii'}^{aa} = 0$) and both trains arrive at the same track *k*, then *i* is allowed to arrive at least $h(i',k,i,k)^{da}$ minutes after train *i′* leaves track *k*, which is expressed by constraint (13). Constraint (14) holds for the reverse order of trains *i*, *i′*.

Constraint (15) states that y_{ij}^{aa} is 1 if train *i* is followed by train *j* at the arrival and both trains travel on the same in-line.

Constraint (16) ensures that each train is always dispatched to exactly one platform track.

The remaining obligatory constraints (17) – (22) specify the definition domains of the variables.

This multiple-criteria optimisation problem was solved using the lexicographic approach, where the objective functions are ranked according to their importance. In the problem at hand, the first objective function (i.e. to meet the timetable) is more important that the second one (i.e. to respect track preferences). This ordering reflects how decisions are currently made in practice. The solution technique consists of two steps. In the first step the problem (1), (3) – (22) is solved giving the best value of the weighted sum of deviations f_1^{best}. Then the constraint

$$c_i \sum_{i \in U} (u_i + v_i) \leq f_1^{best} \qquad (\mathbf{24})$$

is added and the model (2) – (22), (24) is solved. Because both MIP problems are hard and the optimal solutions cannot be found within a reasonable time limit, we decided to implement the local branching heuristic (Fischetti, Lodi, and Salvagnin, 2009) using the general optimisation software *Xpress*.

3. CASE STUDY

The model was verified by using the real data of Prague main station and the timetable valid for the years 2004/2005. Prague main station is a large station that at the given time had 7 platforms, 17 platform tracks and 8 arrival or departure line tracks. According to the timetable 2004/2005, the station dealt with 288 regular passenger trains per a weekday. We could use any timetable for validation, however we used the timetable valid for 2004/2005 because we knew that it was done with some mistakes. We wanted to demonstrate that our model is valid, can detect every possible conflict in the timetable and suggest its solution.

Since the model with 288 trains contains 41279 variables and 595323 constraints, it is not possible to solve it to optimality in a reasonable time. That is why the decomposition of the problem must be done. The planning period (a day) was divided into shorter time periods. They were chosen in such a way so that the morning and evening peak hours were taken as a whole and the rest of the day was divided into shorter periods with approximately the same number of trains. The resulting time intervals can be seen in Table 1.

For every time interval, the mathematical programming model was solved using the lexicographic approach described in the previous section. In case that the exact algorithm (branch and bound method) did not finish in a predetermined computational time (30 minutes) then the local branching heuristic was applied.

The computational experiments were performed for a shorten change train time which is 4 minutes in Prague main stations, as well as for the normal change time (8 minutes). The results for the shortened time are reported in Table 1 and for the normal change time in Table 2.

Table 1. Results of experiments for the decomposed planning period and shortened change time

Time interval [-]	**No. of trains** [-]	**No. of variables** [-]	**No. of cons-traints** [-]	**Value of 1st objective function** [-]	**Value of 2nd objective function** [-]	**Delay on arrival** [min]	**Delay on depart.** [min]	**No. of trains allocated to different platform** [-]
0:00 - 5:00	20	393	6 960	0	20.0	0	0	0
5:00 - 8:00	56	2280	41 965	12	54.6	2	2	5
8:00 - 10:00	45	1576	32 870	13	41.8	0	5	8
10:00 - 12:00	37	1091	21 692	6	35.2	0	2	5
12:00 - 15:00	52	1943	35 804	11	47.9	1	2	9
15:00 - 18:00	54	1993	35 028	6	54.0	0	2	0
18:00 - 24:00	77	3682	57 842	14	75.7	2	4	4

Table 2. Results of experiments for the decomposed planning period and normal change time

Time interval [-]	**No. of trains** [-]	**No. of variables** [-]	**No. of cons-traints** [-]	**Value of 1st objective function** [-]	**Value of 2nd objective function** [-]	**Delay on arrival** [min]	**Delay on depart.** [min]	**No. of trains allocated to different platform** [-]
0:00 - 5:00	20	393	6 960	10	20.0	0	8	0
5:00 - 8:00	56	2280	41 965	12	54.6	2	2	5
8:00 - 10:00	45	1576	32 870	13	41.8	0	5	8
10:00 - 12:00	37	1091	21 692	12	35.2	0	4	5
12:00 - 15:00	52	1943	35 804	11	47.9	1	2	10
15:00 - 18:00	54	1993	35 028	6	54.0	0	2	0
18:00 - 24:00	77	3682	57 842	14	75.7	2	4	4

The results of computational experiments show that the timetable was not correct with regard to safety requirements. There were some trains travelling on conflicting routes concurrently. That is why their desired arriving or departing times could not be kept. Moreover, in some cases the original timetable did not respect desired time passengers need to change trains. However the model respects such connections. The best solution proposed by the model with the shortened change time delays 3 trains at arrival by 5 minutes and

12 trains at departure by 17 minutes in total, and dispatches 31 (11 %) trains to platform tracks different from the planned ones. Departures of 7 trains are postponed by 1 minute and 5 trains by 2 minutes. For the normal change time, 3 trains are delayed at arrival by 5 minutes and 16 trains are delayed at departure by 27 minutes in total (8 trains by 1 minute, 6 trains by 2 minutes, 1 train by 3 minutes and 1 train by 4 minutes). 32 trains are dispatched to platform tracks different from the planned ones.

Other experiments were performed to investigate:

- how decomposition of planning period influence the computational time and the quality of obtained solution within 30 minutes limit for computing,
- how train delays influence the track occupancy plan,
- efficiency of the branch and bound and local branching methods.

4. CONCLUSIONS

In the paper, a mixed integer programming model for the train platforming problem at a passenger railway station is described. The model proposes a track occupancy plan that respects safety constraints for train movements and relations between connecting trains, minimises deviations of the arrival and departure times from the timetable and maximises the desirability of the platform tracks to be assigned to the trains. The model could serve as a planner’s decision supporting tool.

Acknowledgements

This research was supported by the Scientific Grant Agency of the Ministry of Education of the Slovak Republic and the Slovak Academy of Sciences under project VEGA 1/0296/12 “Public service systems with fair access to service” and by the Slovak Research and Development Agency under project APVV-0760-11 “Designing of Fair Service Systems on Transportation Networks”.

REFERENCES

Bažant, M. and Kavička, A. (2009) Artificial neural network as a support of platform track assignment within simulation models reflecting passenger railway stations. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 223 (5), 505–515.

Billionet, A. (2003) Using integer programming to solve the train-platforming problem. Transportation Science 37 (2), 213–222.

Carey, M. and Carville, S. (2003) Scheduling and platforming trains at busy complex stations. Transportation Research Part A 37, 195–224.

Chakroborty, P. and Vikram, D. (2008) Optimum assignment of trains to platforms under partial schedule compliance. Transportation Research Part B: Methodological 42 (2), 169–184.

FICO™ Xpress Optimization Suite [online]. Available from: http://www.fico.com [Accessed 10 October 2011].

Fischetti, M., Lodi, A., and Salvagnin, D. (2009) Just MIP it!. In: Maniezzo, V., Stützle, T., Voß, S. (eds.), Matheuristics. Springer, New York, 39–70.

Zwaneveld, P.J. (1997) Railway Planning - routing of trains and allocation of passenger lines. Ph.D. thesis, Erasmus University Rotterdam, Rotterdam, the Netherlands.

Zwaneveld, P.J., Kroon, L.G., and van Hoesel, S.P.M. (2001) Routing trains through a railway station based on a node packing model. European Journal of Operational Research 128, 14–33.